ECONOMETRICA

JOURNAL OF THE ECONOMETRIC SOCIETY

An International Society for the Advancement of Economic Theory in its Relation to Statistics and Mathematics

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VOL. 7, NO. 4 — OCTOBER, 1939

E C O N O M E T R I C A

Published quarterly by the Econometric Society
The University of Chicago
Chicago, Illinois, U. S. A.

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Subscription Price $7.00 per annum. Single copies $1.75.

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Printed in the U. S. A. by George Banta Publishing Company
Menasha, Wisconsin

ECONOMETRICA
VOLUME 7 OCTOBER, 1939 NUMBER 4

REMARKS ON THE THEORY OF DEPRECIATION

By K.-G. HAGSTROEM

INTRODUCTION

DURING my activity as a life insurance actuary, i.e., from about 1918, I have frequently come back to the question of depreciation and of real or hypothetical insurance against the risk of destruction of mortal property. I have always suspected that in this range of ideas there should be hidden some fundamental connection between prices of consumption commodities and prices of capital goods, some equation of economic equilibrium which could be taken as a basis for a general economic theory. This suspicion was expressed as early as in 1921, in a study of the general economy of the cotton industry in different countries, which was made on behalf of a Swedish committee on customs tariffs. Since that time, a series of economic papers have treated similar problems. In 1925, H. Hotelling published a general theory of depreciation which tried to find the connection I have had in view. Other contributions have been given by R. Frisch, A. J. Lotka, E. B. Kurtz, G. A. D. Preinreich. The last-named author has published an annual survey in ECONOMETRICA, July, 1938.

In the following lines, I shall develop my present view upon the problem. It may perhaps be interesting as an attempt to approach the question from the actuarial side.

THE ACTUARIAL VALUATION

The actuarial valuation of an interest-bearing asset, e.g., a piece of real estate, is a special case of economic planning, or speculation, aiming at a prognosis of future events. It requires a hypothesis on future interest, a forecast on house rent and costs of maintenance, and an estimate concerning the mortality of the estate, i.e., mainly of the building erected on the ground. The economic plan accepted with these hypotheses will also define a rational scheme of depreciation. The complexity of the forecast makes it necessary to introduce reasonable simplifications. The nature of the scheme will thus always to a certain extent remain a matter of taste.

As it is hardly of any use to develop a theory based upon the whole apparatus of the actuary, including an arbitrary function of mortality,

different parameters of loading, etc., we shall choose the simple hypothesis of sudden deaths at the expiration of the lifetime of a building of the kind in question. When it is realized that this lifetime is a statistical variable with a considerable dispersion, it is evident that the economic plan cannot be in accordance with the practice except in a collective way, one building becoming in reality obsolete and ripe for being discarded some time before, another building some time after the hypothetical mean lifetime. Nevertheless, the general plan has to be adhered to, even if it seems to be somewhat unsatisfactory in practice. As a matter of fact, a closer adjustment to the observed conditions could always be obtained by introducing more appropriate functions of mortality, but it is to be observed that all assumptions, however close they may be taken to the statistical data, are necessarily of a stochastical nature and cannot be expected to be valid in a special case. The secrets of the actuarial concept of the "policy value" are evidently applicable here, and they seem often to imply some paradoxes.

An actuarial basis of computation, being a set of forecasts, some of which are decidedly on the safe side, must as a rule give values with a margin of safety, i.e., low values of assets, high values of liabilities. This is especially true as concerns the rate of interest.

We have to consider the value of an asset or the value of the liabilities of a policy as a function of the basis w of calculation, thus mainly as a function of the rate of future interest i_w belonging to the basis w.

The valuation principles may be exemplified by the following treatment of the case of a piece of real estate. Denote by:

- a the value of the ground,
- b the cost of the building, if constructed at the present moment,
- s the total life of a building of this kind,
- t the present age of the building to be valued,
- h the net yield of the estate after deducting expenses and costs of maintenance but no amortization (depreciation).

Letting i denote an arbitrary rate of interest and

$$\delta = \log_e (1 + i), \quad v = \frac{1}{1+i}, \quad \overline{a_{\overline{n}|}} = \frac{1 - v^n}{\delta},$$

we have the following identity:

$$a + b \cdot \frac{\overline{a_{s-t|}}}{\overline{a_{s|}}} + \int_0^{s-t} \left(h - a\delta - \frac{b}{\overline{a_{s|}}} \right) e^{-\delta w} dw \equiv a \cdot v^{s-t} + h \cdot \overline{a_{s-t|}}.$$

Valuing, quite generally, the real estate as the sum

$$V(t) = a \cdot v^{s-t} + h \cdot \overline{a_{s-t|}}$$

of the value of the building ground at the time of obsolescence and

the value of the net output in the meantime, we see accordingly that this value may be interpreted as the present value of the estate according to the depreciation scheme,

$$a + b \cdot \frac{a_{\overline{s-t}|}}{a_{\overline{s}|}},$$

increased by the value of future net output after deduction of an appropriate annuity for future depreciation,

$$\frac{b}{a_{\overline{s}|}},$$

and of the interest on the ground value $a\delta$.

It is unnecessary to remark that the difference under the integral sign may also be negative, but in the normal case it is positive and represents a supervalue according to the rate of valuation i. If i_w is the general rate of valuation of a variable basis w, we may put $i=i_w+j$, j being a risk margin characterizing the real estate in question. If i_0 denotes the rate for which a valuation according to $i=i_0+j$ gives

$$h - a\delta - \frac{b}{a_{\overline{s}|}} = 0,$$

the corresponding value $V_0(t)$ may be considered as a schematic selling value.

The dualism which we have stated in the valuation evidently points to the different views of the *builder* and of the *landlord*, representing seller and buyer of buildings, or buyer of house-building work, and seller of apartment-years, respectively. We shall have to come back to this distinction later.

PROSPECTIVE AND RETROSPECTIVE RESERVE: GENERALIZED SCHEME OF DEPRECIATION

The natural aspect of an actuarial valuation is the prospective view, such as we have exemplified in the preceding paragraph. It is well known that the "policy value" of an insurance admits also a retrospective interpretation. The value $V(t)$ of an asset which we have studied above is, however, no insurance "*reserve*," but its complement. In fact, the general view of depreciation depends upon the introduction of a hypothetical insurance against destruction of the property, and the amount by which the original value of the property has to be diminished at the age of t years represents the policy value or premium reserve of this insurance. In our simple instance, the insurance premium is

$$P = p - \delta = \frac{1}{a_{\overline{s}|}} - \delta = \frac{v^s}{a_{\overline{s}|}},$$

and the policy value can be written

$$W(t) = 1 - V(t) = v^{s-t} - (p - \delta) \cdot a_{\overline{s-t}|} \text{ (prospective reserve)}$$

or $$= (p - \delta) \cdot s_{\overline{t}|} = (p - \delta) \cdot (1 + i)^t \cdot a_{\overline{t}|} \text{ (retrospective reserve).}$$

The premium reserve represents the "loss" through depreciation, and it is seen that in our special case the reserve can be interpreted retrospectively as the value of the total amount of the premiums paid up to the point of valuation. The simple scheme adopted is, in fact, based upon pure savings premiums, all risk being accumulated at the point $t = s$.

One way of making a practical generalization of our simple scheme of depreciation without complicating it considerably would be to introduce a constant force of mortality μ, corresponding mainly to hazardous destruction of the property by fire, inundation, etc., but to retain the total death after s years of remaining properties. The formulae will then, actuarially speaking, be reduced to those of an endowment insurance with constant mortality. Putting as usual

$$\begin{aligned} \delta(\mu) &= \log (1 + i) + \mu, \\ 1 + i(\mu) &= e^{\delta(\mu)} = (1 + i) \cdot e^{\mu}, \\ v(\mu) &= e^{-(\mu+\delta)}, \\ a_{\overline{s}|}(\mu) &= [1 - v(\mu)^s]/(\mu + \delta), \end{aligned}$$

we obtain for the premium

$$P(\mu) = \frac{1 - \delta a_{\overline{s}|}(\mu)}{a_{\overline{s}|}(\mu)} = \frac{\mu + \delta}{1 - e^{-s(\mu+\delta)}} - \delta = \mu + \frac{v(\mu)^s}{a_{\overline{s}|}(\mu)},$$

and for the policy value

$$\begin{aligned} W(t) = 1 - V(t) &= 1 - \frac{a_{\overline{s-t}|}(\mu)}{a_{\overline{s}|}(\mu)} = \frac{e^{(\mu+\delta)t} - 1}{e^{(\mu+\delta)s} - 1} \\ &= 1 - \delta a_{\overline{s-t}|}(\mu) - P(\mu) \cdot a_{\overline{s-t}|}(\mu) \text{ (prospective reserve)} \\ &= \{P(\mu) - \mu\} \cdot s_{\overline{s-t}|}(\mu) = e^{(\mu+\delta)t} \{P(\mu) - \mu\} \cdot a_{\overline{t}|}(\mu) \\ &= e^{(\mu+\delta)t} \cdot \left\{ P(\mu) \cdot a_{\overline{t}|}(\mu) - \int_0^t \mu e^{-\mu w} \cdot e^{-\delta w} dw \right\} \end{aligned}$$

(retrospective reserve, the integral denoting as is easily seen the initial value of the temporary death assurance).

The simplification obtained in this case is a consequence of the simple

hypothesis $\mu =$ constant. In the general case, the differential equation for the premium reserve $W(t)$,

$$dW(t) = \delta W(t)dt + P(\mu)dt - \mu_{x+t}\{1 - W(t)\}dt,$$

may be integrated in two ways. If the savings part of the premium is defined as

$$\pi(t) = P(\mu) - \mu_{x+t}\{1 - W(t)\},$$

or as

$$\omega(t) = P(\mu) - \mu_{x+t},$$

respectively, and the differential equation is multiplied by $e^{-\delta t}$ or D_{x+t}/D_x respectively, we obtain

$$e^{-\delta t} \cdot W(t) = \int_0^t \pi(t) e^{-\delta t} dt$$

or

$$\frac{D_{x+t}}{D_x} \cdot W(t) = \int_0^t \omega(t) \frac{D_{x+t}}{D_x} dt.$$

The latter point of view does not, in the general case when μ_{x+t} depends on t, afford a simplification like that in our case. Our differential equation is

$$\frac{dW(t)}{dt} - (\mu + \delta) \cdot W(t) = P(\mu) - \mu = \frac{1}{a\bar{s}|(\mu)} - (\mu + \delta).$$

The l_x function of this scheme of depreciation is evidently given by

$$e^{-\mu t} = \frac{v(\mu)^t}{v^t} \text{ for } t < s \text{ (and } = 0 \text{ for } t \geqq s).$$

The corresponding frequency function of the deaths is, for $t < s$, $\mu e^{-\mu t}$, thus being a decreasing function which cannot be said to give a rational picture of the reality. But the simplicity of the scheme must make it easy to forgive the unsatisfactory distribution of the hypothetical early deaths, at least for small values of μ. Both components of the distribution of deaths, the continuous branch as well as the discontinuous decrease of the amount $e^{-\mu s}$ for $t = s$, must evidently be considered as rough approximations.

The most interesting feature of the generalized scheme of depreciation, however, is that it may be interpreted as *an annuity scheme with another rate of interest.* This holds true as concerns the *value*, but of course not in respect of the premium.

We start again from our fundamental identity, interpreted now according to the new scheme of depreciation, and we obtain the following identity:

$$a + b \cdot \frac{a\overline{s-t}|(\mu)}{a\overline{s}|(\mu)} + \int_0^{s-t} \{h - (a+b)\delta - a\mu - b \cdot P(\mu)\} \cdot e^{-(\mu+\delta)w} dw \equiv a \cdot v(\mu)^{s-t} + h \cdot a\overline{s-t}|(\mu).$$

Making our valuation according to the formula

$$V(t) = a \cdot v(\mu)^{s-t} + h \cdot a\overline{s-t}|(\mu),$$

we obtain accordingly a value which may be interpreted as the sum of the following two terms:

(1) the depreciated value of the estate

$$a + b \cdot \frac{a\overline{s-t}|(\mu)}{a\overline{s}|(\mu)},$$

which itself is the difference between its value as newly constructed and the premium reserve of the insurance against destruction;

(2) the present value of the future net output

$$\int_0^{s-t} \{h - (a+b)\delta - a \cdot \mu - b \cdot P(\mu)\} e^{-\mu w} \cdot e^{-\delta w} dw,$$

for the remaining lifetime of the building (hence the factor $e^{-\mu w}$), after deduction of costs of maintenance, interest on original vaue $(a+b) \cdot \delta$, and premium for the insurance against destruction of the building $b \cdot P(\mu)$. The subtraction of $a \cdot \mu$ is theoretically unjustifiable but evidently leaves the resulting value on the safe side.

Neglecting for the moment the ground value, the mechanism of depreciation may be described as follows. A building of the cost of construction 1 is amortized with the annuity

$$\frac{1}{a\overline{s}|(\mu)},$$

and the resulting depreciated value is

$$V(t) = \frac{a\overline{s-t}|(\mu)}{a\overline{s}|(\mu)}.$$

The annuity is separated in the following way:

$$\frac{1}{a\overline{s}|(\mu)} = \delta + \mu + \left[\frac{1}{a\overline{s}|(\mu)} - \delta - \mu\right] = \delta + \mu + [P(\mu) - \mu].$$

From the net output is then taken first δ, the pure interest on the original value, next μ as a constant risk premium, and, finally, the savings premium $P(\mu) - \mu$, the accumulated value of which is the premium reserve

$$W(t) = 1 - V(t).$$

The accumulation is made with the inclusion of "hereditary" income, i.e., according to the force of interest $\mu+\delta$, the risk premium being consequently the greater amount μ, not $\mu V(t)$.

The natural way of booking the value of a piece of real estate seems then to be as follows:

Assets	Liabilities
Ground Value.....a	Depreciation (=Premium Reserve of Destruction Insurance). .....$b\left[1-\frac{a_{\overline{s-t}|}(\mu)}{a_{\overline{s}|}(\mu)}\right]$
Cost of Construction.....b	
Supervalue..... $a\cdot v(\mu)^{s-t}+h\cdot a_{\overline{s-t}|}(\mu)-a-b$	

From the net output which is not bound in the reserve of the insurance against destruction, risk premiums are set free in order to meet "claims" of early depreciation. As these must be rare, if the administration has not a gigantic extension, it is recommended to book their accumulated value as a liability and to put on the other side the costs of construction for replacements as they occur.

Identifying the risk premium μ with the annual margin of security denoted above by j, we may observe that a valuation according to the rate of interest prevailing in the real-estate market will deliver no supervalues at all. Only if the balance is based on a smaller rate of interest—which is, however, the rule, if the mass of assets are securities with fixed interest—will the supervalues appear. On the other side, a bonus valuation above the ordinary rate of interest in the real-estate market will result in a negative "supervalue," i.e., in an extra depreciation.

SECOND GENERALIZATION OF ANNUITY DEPRECIATION SCHEME

Proceeding another step in the direction of what might be termed *actuarial interpretism*, we shall give a new modification of our scheme of depreciation.

The new modification starts from the fact that premium payments should preferably be payable in proportion to the remaining value $V(t)=1-W(t)$ of the property, its economic productivity being to a certain extent decreasing as the depreciation proceeds. The differential equation for the premium reserve W will in this case assume the following form, if the constant proportion of V which is taken as insurance premium is denoted by k, the premium itself being consequently $kV=k(1-W)$:

$$dW = k(1 - W)dt + \delta W dt - \mu(1 - W)dt$$

or

$$W' - (\mu + \delta - k)W = k - \mu.$$

Taking μ constant as before, with given values of μ and δ, we have to determine the constant k and the function W. We obtain easily

$$W = (k - \mu) \cdot s_{\overline{t}|}(\mu - k) = (k - \mu) \cdot \frac{e^{(\mu+\delta-k)t} - 1}{\mu + \delta - k}.$$

By putting $t = s$, the total lifetime of the property, we obtain an equation for k:

$$1 = (k - \mu) \cdot s_{\overline{s}|}(\mu - k) = (k - \mu) \cdot \frac{e^{(\mu+\delta-k)s} - 1}{\mu + \delta - k}.$$

The value for k calculated from this relation is a function of δ (or i), μ [or $i(\mu)$], and s. We obtain identically

$$W = \frac{s_{\overline{t}|}(\mu - k)}{s_{\overline{s}|}(\mu - k)} = \frac{s_{\overline{s}|}(\mu - k) - v(\mu - k)^t \cdot s_{\overline{s-t}|}(\mu - k)}{s_{\overline{s}|}(\mu - k)}$$

$$= 1 - \frac{a_{\overline{s-t}|}(\mu - k)}{a_{\overline{s}|}(\mu - k)} = 1 - V.$$

The technique of annuity depreciation will thus be repeated once more, but with a new interpretation. We can write the depreciated value at the time t

$$V = \frac{1 - e^{-(\mu+\delta-k)\cdot(s-t)}}{1 - e^{-(\mu+\delta-k)\cdot s}} = \frac{e^{(k-\mu-\delta)\cdot(s-t)} - 1}{e^{(k-\mu-\delta)\cdot s} - 1}.$$

If we write

$$i_1 = e^{\mu+\delta-k} - 1, \qquad \log(1 + i_1) = \mu + \delta - k,$$

the rate i_1, at which the depreciation annuity is working, will often be negative. In this case, the value V will decrease from 1 to 0 along a curve which will be convex toward the time axis, instead of concave as in our other schemes. The new scheme will require higher depreciations at the beginning of the lifetime of the property. Some specimens of k, compared with the corresponding constant premium $P(\mu)$, are shown in Table 1.

We shall now try to obtain an identity corresponding to the fundamental equation which we have employed above for interpreting the actuarial valuation. Putting $a = 0$ and $b = 1$ we can write this identity

$$h a_{\overline{s-t}|}(\mu) = V + \int_0^{s-t} \left\{ h - \frac{1}{a_{\overline{s}|}(\mu)} \right\} e^{-(\mu+\delta)w} dw$$

$$= \text{depreciated value} + \text{supervalue}.$$

TABLE 1

PREMIUMS FOR INSURANCE AGAINST DESTRUCTION

	Constant Premium $P(\mu)=\frac{1}{\bar{a}_{\overline{s}|}(\mu)}=\delta$ Per Centum			Constant Proportion of Depreciated Value k Per Centum		
I. Constant mortality with $i(\mu)-i=0.5\%$						
$i=$ / $i(\mu)=$	2 % / 2½%	2½% / 3 %	3 % / 3½%	2 % / 2½%	2½% / 3 %	3 % / 3½%
$s=$ 10	9.31	9.08	8.86	29.22	26.52	24.33
20	4.36	4.15	3.96	10.62	9.39	8.33
30	2.74	2.56	2.39	5.69	4.87	4.22
40	1.95	1.79	1.65	3.59	3.03	2.59
50	1.50	1.36	1.23	2.51	2.09	1.76
60	1.22	1.09	0.98	1.87	1.55	1.31
70	1.02	0.91	0.82	1.48	1.23	1.03
100	0.72	0.65	0.60	0.90	0.77	0.67
II. Constant mortality with $i(\mu)-i=1\%$						
$i=$ / $i(\mu)=$	2% / 3%	2½% / 3½%	3% / 4%	2% / 3%	2½% / 3½%	3% / 4%
$s=$ 10	9.57	9.35	9.13	29.71	27.00	24.81
20	4.64	4.45	4.26	11.11	9.87	8.81
30	3.05	2.87	2.71	6.18	5.35	4.70
40	2.28	2.13	2.00	4.08	3.51	3.07
50	1.85	1.72	1.61	3.00	2.57	2.24
60	1.58	1.47	1.38	2.36	2.03	1.79
70	1.40	1.31	1.24	1.97	1.71	1.51
100	1.14	1.08	1.05	1.39	1.25	1.15

In our modified case, we have to start from the differential equation

$$dW = k(1 - W)dt + \delta W dt - \mu(1 - W)dt$$

and write it

$$\frac{d}{dt}\left\{We^{-(\mu+\delta)t}\right\} = k(1 - W)e^{-(\mu+\delta)t} - \mu e^{-(\mu+\delta)t}.$$

Integrating between arbitrary limits t_0 and t_1, we obtain

$$k\int_{t_0}^{t_1}(1 - W)e^{-(\mu+\delta)w}dw + \delta\int_{t_0}^{t_1}e^{-(\mu+\delta)w}dw$$

$$= \left[We^{-(\mu+\delta)w}\right]_{w=t_0}^{w=t_1} + (\mu + \delta)\int_{t_0}^{t_1}e^{-(\mu+\delta)w}dw.$$

TABLE 2

SPECIMEN OF ACTUARIAL VALUATION AT A LOWER RATE

Life of building: 30 years.
Original rates of interest: $i = 3\%$, $i(\mu) = 3\frac{1}{2}\%$.
Valuation rates: $i = 2\%$, $i(\mu) = 2\frac{1}{2}\%$.

	Constant Premium	Constant Proportion of Depreciated Value
Original rates	$h \cdots 0.05344$	$[i_0(\mu - k_0) = -0.78\%]$
	$= \frac{1}{a_{\overline{s}\rceil}^{0}(\mu)} - \delta_0 \cdots 0.02388$	$k_0 \cdots 0.0422$
	$+\delta_0 \cdots 0.02956$	$\delta_0 \cdots 0.02956$
Depreciated values: $V_0(t)$	$= a_{\overline{s-t}\rceil}^{0}(\mu)/a_{\overline{s}\rceil}(\mu)$	$= a_{\overline{s-t}\rceil}^{0}(\mu-k)/a_{\overline{s}\rceil}^{0}(\mu-k)$
$t = 0$	1.0000	1.0000
5	0.8961	0.8167
10	0.7727	0.6404
15	0.6262	0.4708
20	0.4522	0.3077
25	0.2455	0.1508
30	0.0000	0.0000
Depreciated values: $V(t)$	$= a_{\overline{s-t}\rceil}(\mu)/a_{\overline{s}\rceil}(\mu)$	$= a_{\overline{s-t}\rceil}(\mu-k)/a_{\overline{s}\rceil}(\mu-k)$
$t = 0$	1.0000	1.0000
5	0.8803	0.7599
10	0.7448	0.5555
15	0.5916	0.3815
20	0.4182	0.2334
25	0.2220	0.1073
30	0.0000	0.0000
Total values: $V(t) + \Delta(t)$	$= h a_{\overline{s-t}\rceil}(\mu)$	$= \frac{k_0}{k} \cdot \frac{a_{\overline{s-t}\rceil}(\mu-k)}{a_{\overline{s}\rceil}(\mu-k)} + \left(\delta_0 \frac{k_0}{k} \delta\right) a_{\overline{s-t}\rceil}(\mu)$
$t = 0$	1.1324	1.0570
5	0.9969	0.8411
10	0.8435	0.6468
15	0.6699	0.4695
20	0.4735	0.3050
25	0.2514	0.1496
30	0.0000	0.0000
Super-values: $\Delta(t)$		
$t = 0$	0.1324	0.0570
5	0.1166	0.0812
10	0.0987	0.0913
15	0.0783	0.0880
20	0.0553	0.0716
25	0.0294	0.0423
30	0.0000	0.0000

With $t_0=t$ and $t_1=s$ (prospective case) we obtain after dividing by a discount factor

$$k\int_0^{s-t}(1-W)e^{-(\mu+\delta)w}dw+\delta\bar{a}_{\overline{s-t}|}(\mu)=1-W(t)=V(t).$$

The corresponding equation in the constant-mortality endowment case is

$$P\cdot\bar{a}_{\overline{s-t}|}(\mu)+\delta\cdot\bar{a}_{\overline{s-t}|}(\mu)=V(t).$$

In either case, the value of future premiums and interest, payable during the lifetime of the building, is identified with the depreciated value of the property. In the latter equation we have $P+\delta=1/\bar{a}_{\overline{s}|}(\mu)$, which makes it evident that, if $h=1/\bar{a}_{\overline{s}|}^{0}(\mu)$ corresponds to a higher rate of interest i_0, the expression $h\bar{a}_{\overline{s-t}|}(\mu)$ must contain a supervalue. In the same way, we can prove that the value, according to the rate of valuation i, of premiums and interest in the generalized scheme, calculated with a higher rate of interest i_0, viz.,

$$k_0\cdot\int_0^{s}(1-W)e^{-(\mu+\delta)w}dw+\delta_0\cdot\bar{a}_{\overline{s}|}(\mu),$$

will be positive, and consequently represents a supervalue in this valuation. In fact, we have

$$k\int_0^{s}(1-W)e^{-(\mu+\delta)w}dw=1-\delta\bar{a}_{\overline{s}|}(\mu),$$

and the value in question is therefore

$$\frac{k_0}{k}\left[1-\delta\bar{a}_{\overline{s}|}(\mu)\right]+\delta_0\bar{a}_{\overline{s}|}(\mu),$$

or the sum of two positive numbers. Starting from values k_0 and δ_0 corresponding to a high market rate, we can write

$$k_0\int_0^{s-t}(1-W)e^{-(\mu+\delta)w}dw+\delta_0\bar{a}_{\overline{s-t}|}(\mu)$$

$$=(k_0-k)\int_0^{s-t}(1-W)e^{-(\mu+\delta)w}dw+(\delta_0-\delta)\bar{a}_{\overline{s-t}|}(\mu)+V.$$

This is the fundamental identity in our modified scheme. The first member may also be written

$$\frac{k_0}{k}\left\{1-\delta\bar{a}_{\overline{s-t}|}(\mu)-W(t)\right\}+\delta_0\bar{a}_{\overline{s-t}|}(\mu)$$

$$= \frac{k_0}{k} \left\{ V(t) - \delta a_{\overline{s-t}|}(\mu) \right\} + \delta_0 a_{\overline{s-t}|}(\mu)$$

$$= \frac{k_0}{k} \cdot \frac{a_{\overline{s-t}|}(\mu - k)}{a_{\overline{s}|}(\mu - k)} + \left(\delta_0 - \frac{k_0}{k} \cdot \delta \right) a_{\overline{s-t}|}(\mu).$$

The modified scheme perhaps seems to be unnecessarily complicated. For practical purposes, the simple constant-mortality endowment scheme seems to be quite satisfactory.

THE PROBLEM OF REPLACEMENT

The economic plan of depreciation, as developed in the preceding paragraphs, is independent of the rate according to which new constructions are effectuated, i.e., as in all forms of rational insurance, of the fluctuations in the number of *new policies*. The development of the *total stock* of insurance may have its special interest, e.g., for questions of central costs, but these problems must not have any vital importance for the economy of the insurance. In a stationary stock of insurance policies, where claims are exactly counterbalanced by new contracts, some questions of costs take an especially simple form, but it would be dangerous to base the economy of the insurance on this quality, which can as a rule only be an ephemeral phenomenon.

It is true that the machinery of a mill or the buildings of a house-owning company or the cars of a transport company form as a rule a stationary equipment. But this is true only if comparatively short periods of time are considered. Modernization, trade cycles, adverse price conditions, to mention only a few prominent features, may cause violent fluctuations in the stock of mortal property. It is just the same with the stock of active workers in a factory or in a transport administration. They form what may be denoted as a population, which may seem to be remarkably constant. The distribution of ages is, however, often far from constant, owing to the fact that a great many young workmen were hired at the beginning of the enterprise. Only in very old plants may the distribution of ages be uniform. But every extension of the industry will quickly alter this status, if mainly young employees are added to the old stock. Fluctuations in the sum of policy values of the pension liabilities will then be unavoidable, but these fluctuations are only *the visible result of an even distribution of the pension costs* in proportion to the cost of active labour.

In the same way it is through fluctuations in the sum of the depreciated values $V(t)$, or in its complement $W(t) = 1 - V(t)$, that inequalities in the new construction are reflected, whereas the *depreciation cost*, i.e., the premiums for destruction insurance, are distributed as evenly as

possible over the active years of the property. There seems not to be any reason at all for trying to make the capital values more constant and letting the depreciation costs vary. Every attempt to move those costs forward, because of present financial stress, seems to be unsound, or at least a suspicious sign of weakness.

Starting from an arbitrary "bell-shaped" function of distribution of the dates of death around the mean lifetime of the property,

$$\rho(t)dt = \mu(t)e^{-\int_0^t \mu(w)dw}dt,$$

it is possible to derive the "function of replacement" at any later date as the result of a functional operation on $\rho(t)$ or on $\mu(t) = \rho(t)/\int_t^\infty \rho(w)dw$. In some cases, the successive phases of new construction will tend towards a stationary limit. In our simple amortization scheme, the replacement takes place discontinuously, the total equipment being replaced after the expiration of s years, the constant lifetime of a single property.

Several authors have studied in detail the statistics available concerning the length of the lives of machines and other property and the resulting replacement functions. It may certainly be of interest to study the real distribution function $\rho(t)$ or its actuarial transformation $\mu(t)$ for different classes of property. As a matter of fact, however, the economic consequences of small deviations from simpler hypotheses concerning the lifetime of mortal property do not seem to be important, as compared with the great uncertainty of other elements of the prognosis, especially the future rate of interest.

THE STOCHASTICAL CONDITIONS OF EXCHANGE

Instead of basing the abstract theory of exchange, as is usually done, upon the notion of ophelimity, I have proposed to create a *stochastical theory of exchange*, a sketch of which was published in ECONOMETRICA, Vol. 6, January, 1938, p. 40. We shall develop this idea for the case of capital goods which we have been considering above.

The *builder* B and the *landlord* L are calculating their respective *subjective values* of the real estate, B starting from a formula

$$a + b \cdot \frac{\mathbf{a}_{\overline{s-t}|i}(\mu)}{\mathbf{a}_{\overline{s}|}(\mu)},$$

to which is added his profit, and L starting from the formula

$$a \cdot v(\mu)^{s-t} + h \cdot \mathbf{a}_{\overline{s-t}|}(\mu),$$

his profit being the difference between that value and the selling-price. Since, however, a, b, h, i, μ, s are only estimates, and often very rough

ones, both subjective values are statistical variables with considerable deviations or "errors of observation." Denote by $\beta(z)$ and $\lambda(z)$ the distribution functions defining the subjective values of B and L respectively, we obtain for the probability that both buyer and seller shall sign a contract based upon a certain value V:

$$\phi(V) = \int_{-\infty}^{V} \beta(z)dz \cdot \int_{V}^{\alpha} \lambda(z)dz.$$

Putting

$$B(z) = \int_{*}^{\alpha} \beta(z)dz, \quad \Lambda(z) = \int_{*}^{\alpha} \lambda(z)dz,$$

we obtain for the function $\phi(z)$:

$$\phi(z) = \Lambda(z) \cdot [1 - B(z)].$$

Denoting by $x_0 \cdots x_1$ the interval in which the mass of probability $\beta(z)$ is concentrated, by $y_0 \cdots y_1$ the corresponding interval for the mass $\lambda(z)$, the general practical case will be that these intervals have a common part. (In the special case treated in ECONOMETRICA in 1938, both intervals were $-\infty \cdots +\infty$.) The point of maximum probability, $z=V$, will be situated in the common part $x_0 \cdots y_1$ or $y_0 \cdots x_1$ of the mass intervals. From a practical point of view, the extreme boom case, where $x_1 < y_0$, and the extreme depression case, where $y_1 < x_0$, may certainly be neglected. It is then easily shown that the point of maximum probability will be situated in the middle of the common interval, if the mass distributions $\beta(z)$ and $\lambda(z)$ are approximately constant in this interval.

The mass distribution $\phi(z)$ will represent the connection existing between the elements entering into the computation. For any special value $z=V$ we have an equation between the elements, and to each equation corresponds the probability of its realization $\phi(V)$.

The classical theory of exchange is based upon the idea that each party will try to realize the greatest profit available and that only this point of view will define the price. The complexity of the developments to which this fundamental idea has brought economic science are far from promising for the possibility of basing world economy on rational principles. The stochastical theory of exchange represents an attempt at restatement, unripe and inconsequent it may be, but perhaps in one or other shape necessary.

When an engineer builds a bridge, he does not construct it so that the absolute minimum of material is consumed in order to obtain the solidity just needed. He is working with wide margins. If he does not,

he is to be blamed. In the same way, I suppose that very few business transactions in this world are made in accordance with the ideal equations of the mathematical theory of exchange. There are so many human factors connected with the construction of a new house, e.g., that the principle of maximum profit will not pay in the long run. Is it not characteristic of the application of the classical theory in this field, that the main thing obtained by it seems to be some conclusion about the scrap value? According to Preinreich, the search for the true depreciation method has apparently ended in a failure. It seems to me that the reason for this statement must be sought in the principles which I have tried to apply.

Stockholm

A THREE-DIMENSIONAL REPRESENTATION OF THE FACTORS OF PRODUCTION AND THEIR REMUNERATION, MARGINALLY AND RESIDUALLY

By Irving Fisher

A half century ago, before mathematics had invaded economics, one of the great questions in the problem of functional distribution was: "Which is the residual claimant?"

Perhaps the most orthodox view was that land rent is the residual, being what was left after wages, interest, and profits had been taken out, each of these three being determined by its own special law.

Walker, however, insisted that wages were the residual element. Rent of any land was determined by its excess of productivity over the no-rent land; profits likewise by the excess of an entrepreneur's productivity over a no-profit entrepreneur, and interest by its own special law of abstinence, making wages simply what was left.

Others seemed to make interest residual while the practical man was content to regard profits as the obvious residual.

Later, as the marginal theories developed and differential quotients were employed, it came to be believed that each of the four could be regarded as marginally determined not by entirely separate laws but by acting and reacting on each other "like so many balls in a bowl."

Later still the question arose whether this was mathematically possible. Could all the factors of production be remunerated on the marginal principle and yet the sum total of such remunerations agree with the total product and leave no residual, positive or negative?

Wicksteed showed that, if each of the factors was subject to a law of diminishing returns, then the sum of all the returns would check properly with the total product provided, however, and only provided, that these factors were, when taken together as a group in any fixed proportions, subject to a law not of diminishing returns but of constant returns. Wicksteed used mathematical analysis. Later Professor T. N. Carver worked out the same result arithmetically.

The present paper presents the same picture geometrically. For this purpose the ordinary geometrical picture in which marginal rates are represented by lines is not as well suited as another construction whereby each marginal rate is represented by the slope of a curve. This slope method is the same in principle as that used in the theory of prices by Auspitz and Lieben.

In the figure shown on the left in the model let the abscissa L represent one factor, say labor, and let the ordinate P represent the total

product corresponding to the various values of L, every other factor being assumed unchanged.

When $L=0$, we assume that the total product P will be zero. The curve thus starts at the origin and increases continuously in height with each successive increment of L. The total wages, that is the product of L multiplied by the slope or rate of wages, will evidently be represented by W, the heavy line. This leaves a residual R, to include the remunerations of all the other factors.

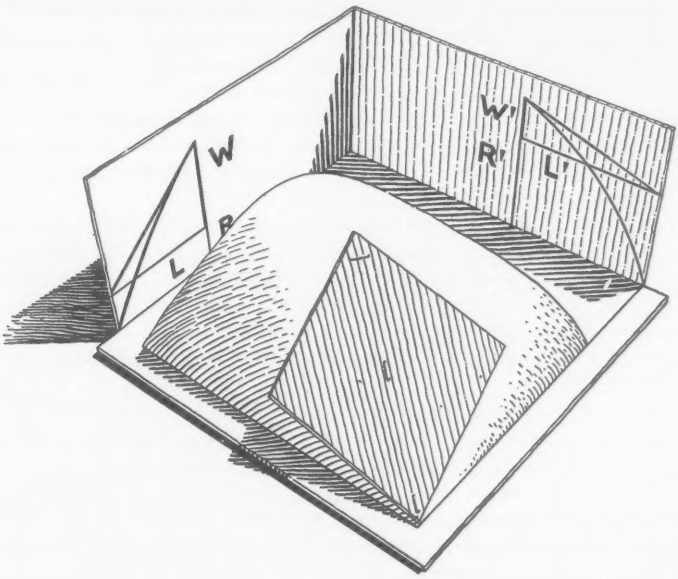

FIGURE 1.

The same diagram could of course be used for any other factor of production, even land, assuming that, analogously to the labor market, successive additional units of homogeneous land are available at a market price.

In that case, the slope, instead of representing wages, would represent the rent of land. Land rent would then be represented by the heavy line previously labelled W while R would represent, as residual, the remuneration of all the remaining factors, including labor.

The next question is: Whatever L stands for, can R, and the parts into which R may further be split, be determined marginally just as was W, and, still, when added to W, make up the total product? Light

on this question will be thrown as soon as we add another dimension to our diagram.

In the resulting three-dimensional model the vertical ordinate, which we shall now call "altitude" still represents the total product P and the abscissa which we shall now call "longitude," extending from the origin toward the right, will represent the variable "labor," L, just as originally supposed.

But we now have, in the remaining dimension, a new variable extending back or away from the observer. This will be called "latitude" and represents all the remaining factors of production. For the present we assume these remaining factors to be all congealed into one group, a composite, L', made up of certain fixed proportions of land, and implements (or rather of each specific variety of these general categories) as well as of every other factor involved. Such a congealed composite is here assumed in order that we may get all our factors of production into three dimensions.

We thus have a surface which is the locus of all points having the three co-ordinates L, L', and P, called respectively, longitude, latitude, and altitude. It must be a *conical* surface, with its apex at the origin. No other surface will meet our two requirements: (1) that the remunerations W and W' of the factors, L and L' respectively, are determined by the marginal principle and (2) that, at the same time, the sum $W+W'=P$.

Evidently L's rate of wages, formerly shown as the slope in the left-hand figure, is now shown as the slope of this surface, namely the slope of a tangent line drawn in a vertical plane running longitudinally.

Similarly the rate of remuneration of L' is the slope of another tangent line drawn through the same point in the surface but in another vertical plane at right angles to the first, i.e., running latitudinally. These two slopes representing W and W' respectively are, of course, the partial derivatives of P with respect to L and L' respectively.

The first-mentioned plane, that containing the tangent expressing W, the rate of wages, cuts our surface in a curve, producing the above-described figure projected on the left-hand screen of the model, and pertaining to L and W. The other plane cuts that surface in a similar curve pertaining to L' and W'.

Now these two tangent lines determine a tangent plane.

If the surface is conical, with apex at origin, this tangent plane, wherever situated, must evidently pass through the origin and so must always contain an "element" of the cone, i.e., a straight line drawn in the surface from the origin to the original point of tangency—a "radius vector."

The only part of this tangent plane which concerns us is the paral-

lelogram formed by the two original tangent lines and the intersections of the tangent plane with the two vertical planes formed by the axes. Evidently the four-sided figure so formed is a parallelogram since the tangent lines run parallel to those planes. The diagonal of this parallelogram is the element of the cone, or radius vector, just mentioned.

The model will enable us to see—literally to see with the eye and as a consequence of the "conical" assumption—that $W+W'=P$ and also that, without that assumption, in general $W+W' \neq P$.

If we project backward, on to the back screen, the first curve formed by a section cut through the given point longitudinally and so parallel to said screen, we shall again reproduce the original LP curve and tangent.

In this projected figure W evidently registers the difference in level between the two corners of the parallelogram there projected, while the residual R registers the remaining difference of level down to the base plane.

In precisely the same way the projection on the right-hand screen shows W' as the difference in level between the two corners of the parallelogram there projected while the residual R' shows the remaining difference of level down to the base plane.

The question as to residuals is now reduced to this: Is $R=W'$ or again is $R'=W$ or again is W on the left-hand screen plus W' on the right-hand screen equal to P?

The answer is, obviously, yes, as will be seen by a glance at the parallelogram. Since its opposite sides are equal, the difference of level between the ends of one side must be exactly the same as the difference of level between the ends of the opposite side.

We can best follow these differences in level by imagining a fly, starting from the given point of tangency, to walk half way around the parallelogram to the origin. He can travel in either direction. If first he travels in longitude, i.e., leftward as seen by the observer, his descent along the first side of the rectangle means a drop in level equal to W. Then, turning on his staircase, he walks in latitude, i.e., facing us, down the next side. This brings him to the origin. In descending this second flight he drops down by R. But since this drop (R) of his second flight must exactly equal the drop which, had he started the other way, he would have experienced in passing down his first flight, namely a drop of W', it follows that $R=W'$.

Likewise $R'=W$; also, since $W+R=P=W'+R'$, it follows that $W+W'=P$.

But this result (that $W+W'=P$) will not hold true generally for any surface other than a conical one with apex at origin; for no other surface will admit of *all* tangent planes passing through the origin.

Thus, suppose the surface is convex like the surface of a derby hat and that it is so situated that the tangent plane, instead of passing through the origin, passes above it, intersecting a vertical through it at a point R'' above it. Then evidently $P = W + W' + R''$. That is, the fly after walking down W and then down W' (or W' and then W) in order to get all the way down to the origin will still have to climb down a residual distance, R'', on an imaginary vertical flagpole standing upright over the origin.

In other words, if the surface is *not* conical with apex at origin, then W and W' can *not* both be marginally determined without leaving some residual—except of course, in special cases.

To say that our locus is a conical surface is simply to say, in geometrical imagery, that, if *all* the factors of production jointly producing a certain product are doubled, the product will be doubled; and if tripled, then tripled; etc.—i.e., the product will vary according to a law of proportionate return on all factors combined in a given proportion, as distinct from the laws of diminishing or increasing returns applicable to any and all the individual factors.

In particular, the law of diminishing returns may apply not only as it has commonly been applied, namely to capital and labor in relation to land, but also to land in relation to capital or to labor. Also when the rate of remuneration of any one factor is worked out by one of these various methods, it will give the same result as if worked out by any other—always provided however, and only provided that the "conical" or proportionate law holds true of all factors combined.

It may also be shown that every curve resulting from a section of the cone by a plane parallel to the first plane (the longitudinal one of Figure 1, showing L and W) must be exactly similar to every other longitudinal section; likewise: Every latitudinal section must be exactly similar to every other latitudinal section; also: If one of these sets of curves, say the LW set, is according to a law of diminishing returns, then the other ($L'W'$) set will also be according to that law.

The first of these propositions, namely that parallel conic sections are similar is, of course, a familiar one and needs no demonstration here.

The last proposition, namely that if one set of similar sections is according to a law of diminishing returns, i.e., is convex, the other must be, may be made evident by observing an infinitesimal portion of the model at or near the original point of tangency.

The convexity of a curve such as that of the first figure means that any point in the tangent line infinitesimally near the point of tangency lies *above* the curve. This curve is that caused by a longitudinal section of the cone. The same applies to the other, or latitudinal, section.

What we wish to prove is that if one of these two curves is convex, (and so also all others made by sections parallel thereto) the other must be convex (and so also all others made by sections parallel thereto).

Now conceive our parallelogram, instead of extending all the way to the origin, to shrink into an infinitesimal one at the original point of tangency and consider, thus in miniature, the two sides over which the fly was supposed to walk. The point from which he started was the original point of tangency and the point at which he stopped was that diagonally opposite (now only infinitesimally distant). This stopping point is also a point of tangency—tangency of a latitudinal tangent, just as the starting point was of a longitudinal one.

In passing over these two sides—both tangents—the fly turned, at a halfway station. On this corner, in the miniature, we now fix our attention.

Evidently that corner lies vertically *above* the conical surface. Moreover the particular point in that surface which lies vertically below that corner is a point common to the two sections of the cone, namely the two vertical sections containing the two respective tangents.

The fact that this common point lies *below* the corner proves the convexity of *both* the curves passing through that point, just as the convexity of the curve of the first figure is proved by the fact that points in the curve infinitesimally near the point of tangency lie *below* the tangent. If one of the two curves is convex the other must be; for it is the same common point which tests both curves by being below the same common point in both tangents. If it is below one it must be below the other.

It follows from what has been said that, if we start with only one solitary curve, say that of the left-hand screen of Figure 1, and then generate a cone by swinging around it a radius vector, we shall have constructed the entire model, just as geologists reconstruct an entire fossil from one bone.

Translated into nongeometrical terms, this means, among other things, that, (1) if we know the law connecting L and W for only one value of L' and (2) if we know that a law of proportionality applies to two factors combined, we can derive the complete law in these two factors.

If we were to apply our three-dimensional analysis to less comprehensive co-ordinates by leaving out some factor or factors, we may be sure that our surface will not be the conical one represented above. It would presumably be more like the surface of the derby hat representing not a law of proportionality but a law of diminishing returns, not only for each of the two variables separately but also for any composite of them both. That is to say, if a vertical plane be passed through the

origin it will not cut the surface in a straight line as it necessarily does when the surface is a cone, but in a convex curve. This curve represents the diminishing return from successive increments of all the factors represented in the model—incomplete by hypothesis.

But since this can no longer be shown by ocular demonstration as was done with the parallelogram the result must be shown by the use of algebraic symbols—ordinary mathematical "analysis"—as follows:

If the law of proportionality holds true as to all the factors combined in fixed ratios, and if these factors are, for convenience, now represented by L, L', L'', etc., we have, analogously to the former case of only two variable factors,

$$P = \sum lL,$$

when l, l', l'', etc., are coefficients which are constant for any given straight line through the origin. Then it may be shown as before that $\partial P/\partial L = l$, etc., so that the equation becomes $P = \sum(\partial P/\partial L)L$.

That is, assuming proportionality for all factors combined however numerous, the product P may be fully determined by the marginal principle without leaving any residual. Also, under any other assumption than proportionality, a residual must emerge.

We have implicitly made the usual assumptions in marginal analysis such as: that there is continuity; that every factor including land is available in infinitely divisible quantities; that each factor is homogeneous; that it bears a fixed market price determined by competition; that each competitor acts independently and is so small a part of the entire market as to be unconscious of any influence on price; and that each person has all the knowledge needed to act in his own best interest.

But we still have the question: Is it, or is it not, true, when we step out of the vacuum chamber of pure theory and put our feet on the ground of the real world, that the total product doubles with a simultaneous doubling of all the factors of production?

If not, then it is impossible that W and W' can both be determined marginally.

While arguments could be offered to indicate that this proportionality law is probably fulfilled approximately, no one would, I think, have the hardihood to say or think that it must of necessity always be true in terms of real as distinct from hypothetical conditions.

From this it follows that we must keep a place in any system for residuals.

Practically, of course, we always encounter residuals in the form of a balancing item—profits.

But there is no need here to argue the question; for this article is confined to describing our three-dimensional representation. And such

a representation may be used to exhibit residuals as well as no residuals.

This representation has a double usefulness. It helps the classroom student to see, literally to see with his eyes, the truth of the following four propositions, which are not easy for him to see in any other way.

(1) A law of diminishing returns may exist for each separate factor of production while at the same time a law of proportionality may hold true for all the factors taken together in a composite.

(2) If this is the case, the remuneration of each factor according to its marginal contribution to the product will leave no residual unaccounted for, but if this is not the case there must be a residual.

(3) Given a law of proportionality for all factors combined and a law of diminishing returns for one factor or group of factors alone, it follows that the remaining factors grouped in a composite must be subject to a law of diminishing returns; moreover that law must be of a certain specified type as determined by the hypothesis just stated.

(4) Under the same conditions either of the two laws of diminishing return applies without change, no matter what the assumed magnitude of the other co-ordinate factor or group may be.

For elementary students not familiar with the concepts and notations of the calculus it is only necessary to consider the tangent lines as lines drawn through the tops of ordinates one unit apart.

The other usefulness of this three-dimensional construction is in helping clarify the thoughts of economic thinkers themselves; for correct visual pictures usually yield the clearest concepts.

Yale University
New Haven, Connecticut

THE MAXIMIZATION OF PROFITS OVER TIME WITH CHANGING COST AND DEMAND FUNCTIONS[1]

By A. SMITHIES

THE PURPOSE of this paper is to analyze the behavior of an entrepreneur under conditions where the "static" demand and cost functions that confront him are changing over time. The device, that is sometimes adopted,[2] of assuming that an entrepreneur equates marginal cost with marginal revenue at every point of time is inadequate since it assumes that he lives a hand-to-mouth existence, producing at every point of time the amount that he sells at that time. Although there are cases where this assumption is justified, it is a commonplace that there are important cases when, say, prices and costs are rising, or when costs only are rising, where an entrepreneur produces for inventory with a view to selling at some subsequent date. An analysis of the behavior of an individual under dynamic conditions should take account of this possibility, and fortunately the necessary amendment to the static analysis can be very simply made.[3]

In the first section of this paper, I shall give a nonrigorous nonmathematical account of the matter. In the second, the same general conclusions will be reached by mathematical methods, and in the third the analysis will be applied to a special case where the demand and cost functions are of known shape.

I

I shall assume, to start with, that an entrepreneur has emerged from a depression with no inventories, either of finished products or raw materials, on hand, and that he is on the threshold of a period during which he expects both prices and costs to be rising for a finite period, and, further, that he thinks he can predict with certainty the behavior of his demand and cost schedules over the period. The length of the period will be determined by the extent of his foresight, and it seems reasonable to suppose that he will play safe by planning to conclude the period with no inventories on hand. I shall also assume that his inventories consist exclusively of his finished product.

[1] In the preparation of this paper I am greatly indebted to Professor G. Y. Rainich of the Mathematics Department of the University of Michigan, and also, inevitably, to Professor Evans' *Mathematical Introduction to Economics*, New York, 1930.

[2] Cf. R. F. Harrod, *The Trade Cycle*, Oxford, 1936, p. 75.

[3] I am not suggesting that this is the only amendment necessary. My purpose is the modest one of suggesting an extension of the use of the analytical tools that have now become part of every economist's equipment.

It is not a matter of indifference to an entrepreneur at what time during the period a given amount of revenue is received or a given amount of costs are incurred. He will seek to maximize his profits as at some particular point of time. It may be the beginning, the end, or some time during the period. If he chooses the beginning, he will discount all prospective returns and costs so as to value them at that point, or he may consider returns and costs accumulated to the end of the period. But provided the rate of interest used is constant over time, the results of his calculations will be the same, whichever point is chosen. For the sake of slight convenience in the mathematical analysis I shall assume that he makes his calculations as at the beginning of the period, and that both returns and costs are discounted at the same given rate.

If, for the sake of simplicity, we are content to assume that the only costs of holding inventories are interest costs, the process of discounting will automatically deal with the inventory problem. For if a batch of goods is produced at a given time with a view to sale say six months thence, the costs of holding the goods for the six months will be accounted for by the discounting of the prospective returns for six months longer than the prospective costs.

Our problem can now be reformulated: It is required to determine production and sales over time in such a way that the excess of the present value of the prospective sales over the present value of prospective costs is a maximum, subject to the condition that the total quantity sold is equal to the total quantity produced during the whole period.

Once the problem is stated in this way, its solution becomes fairly obvious. With the limitation that an entrepreneur cannot sell what he has not yet produced, he is at liberty to sell any quantity at any time so as to maximize his revenue. From this it follows that he will so distribute his sales as to equalize the discounted marginal revenues of the goods sold at all times during the period; since by doing this he will have a greater total present expectation of return than with any other distribution of sales. Similarly, in order to minimize costs of his entire output, he will distribute his production so as to equalize discounted marginal costs throughout the period.

Further, for the net revenue over the whole period to be maximum, it is necessary that the present value of the marginal costs of the whole output is equal to the present value of the marginal revenue from sales over the whole period. But these are equal respectively to the equal discounted marginal costs of the goods produced and marginal revenues from the goods sold at every point of time during the period. Hence we have the solution to the problem: In order to maximize his net

revenue over the whole period, an entrepreneur will so distribute his production and sales as to make the discounted marginal revenue from sales and the discounted marginal cost of goods produced at every point of time both constant and equal. It should be noted that if we chose the end of the period, we should merely have to substitute the word "accumulated" for the word "discounted" in the formulation. The quantity of inventories held at any time will be merely the difference between total production and total sales up to that date.

It must be noted that this solution may require the impossible, namely that total production should not at least keep pace with total sales throughout the period. That is merely another way of saying that it is not always profitable to produce inventories for future sale during a period of change. In such a case, the entrepreneur will maximize his revenue by the "hand-to-mouth" method and will equate marginal costs with marginal revenue at every point of time.

Without knowledge of the functions involved and their rates of change through time it is difficult to formulate conditions necessary for the solution to be of the "inventory" or the "hand-to-mouth" type. Dominant considerations, however, will be the relations between the rates of change of the demand and cost schedules through time, and the rate of discount, and the shapes of the demand and cost schedules. Even though both the demand and the cost schedules are rising, the rate of interest may be sufficiently high to make the cost of holding inventories prohibitive. And if the demand schedule is rising while the cost schedule is falling, the rate of discount may be sufficiently low to make profitable the production of inventories when demand is fairly low in order to avoid increasing cost at a later time when demand has strengthened. Where the demand and cost schedules are both falling through time, it seems hardly likely that it will be profitable to produce for inventories, but it is theoretically possible given a sufficiently low rate of discount together with strongly increasing costs.[4]

II

Let $S=S(t)$ be the annual rate of sales at time t,
$R=R(t)$ be the annual rate of production at time t,
r be the annual rate of interest, which is assumed constant,
$C=C\ (R, t)$ be unit cost at time t, if the annual rate of production is R,
$P=P\ (S, t)$ be the unit price at time t, if the annual rate of sales is to be S.

[4] This, of course, is quite a different problem to the accumulation of inventories due to an *unexpected* fall in demand.

C and P, together with their derivatives, are known functions, while S and R are to be determined as functions of time.

The problem then is to find S and R so as to maximize total profits

$$\pi = \int_{t_0}^{t_1} (PS - CR)e^{-rt}dt, \tag{1}$$

subject to

$$\int_{t_0}^{t_1} (S - R)dt = 0. \tag{2}$$

This is a problem in the Calculus of Variations.[5] Write

$$S + \lambda\sigma \quad \text{for} \quad S$$

and

$$R + \lambda\rho \quad \text{for} \quad R,$$

where σ and ρ are arbitrary functions of t, and λ is a parameter, $0 \leqq \lambda \leqq 1$. Substituting these values in (1):

$$\pi = \int_{t_0}^{t_1} [(S + \lambda\sigma)P(S + \lambda\sigma) - (R + \lambda\rho)C(R + \lambda\rho)]e^{-rt}dt.$$

Now π is a function of λ, and for S and R to be the required extremal functions it is necessary that π should become a maximum where $\lambda = 0$. Differentiating partially with respect to λ and equating with zero, where $\lambda = 0$,

$$\left(\frac{\partial\pi}{\partial\lambda}\right)_{\lambda=0} = \int_{r_0}^{t_1} \left[\left(P + S\frac{\partial P}{\partial S}\right)\sigma - \left(C + R\frac{\partial C}{\partial R}\right)\rho\right]e^{-rt}dt = 0. \tag{3}$$

From (2)

$$\int_{t_0}^{t_1} (\sigma - \rho)dt = 0. \tag{4}$$

$\sigma = \rho$ will satisfy (4) hence (3) must hold for the special selection $\sigma = \rho$. Since these functions are arbitrary, this requires

[5] This problem differs from those to which the technic of the Calculus of Variations is usually applied, since no derivative appears under the integral sign. This distinguishes the present problem from those dealt with by Evans, *op. cit.*, Chapter XIV; Roos, "A Mathematical Theory of Competition," *Amer. Jour. Math.*, Vol. 47, 1925, p. 163; and Tintner, "Monopoly over Time," ECONOMETRICA, Vol. V, 1937, p. 160. However, Professor Hotelling has applied methods closely analogous to the present to the optimum rate of consumption of exhaustible resources. See his "Economics of Exhaustible Resources," *Journal of Pol. Econ.*, Vol. 39, 1931, p. 137. Cf. also Professor Evans' analysis of discontinuous production, *op. cit.*, p. 156.

$$P + S\frac{\partial P}{\partial S} = C + R\frac{\partial C}{\partial R}. \tag{5}$$

Hence (3) may be rewritten

$$\int_{t_0}^{t_1}\left(P + S\frac{\partial P}{\partial S}\right)(\sigma - \rho)e^{-rt}dt = 0. \tag{6}$$

(6) can be satisfied for all shapes of σ and ρ only by

$$\left(P + S\frac{\partial P}{\partial S}\right)e^{-rt} = \Gamma, \text{ a constant.} \tag{7}$$

Similarly

$$\left(C + R\frac{\partial C}{\partial R}\right)e^{-rt} = \Gamma. \tag{8}$$

That is the discounted marginal revenue from sales and the discounted marginal cost of goods produced at all points of time are constant and equal.[6]

The value of Γ can be obtained by solving (7) and (8) for S and R, substituting in (2), integrating, and solving for Γ. Hence the problem is completely determinate.

For the solution to be economically possible we must have total output at least keeping pace with total sales, i.e.,

$$\int_{t_0}^{t} Sdt \leqq \int_{t_0}^{t} Rdt, \qquad t_1 > t > t_0. \tag{9}$$

With functions of any shapes that are likely to occur, it is probable that condition (9) will be either fulfilled or not fulfilled for all values of t. Assuming this to be so, if the above solution does not satisfy (9), it is evident that the greatest value of π that is economically possible will be obtained by solving the problem subject to the further restriction that sales and output are equal at every point of time,[7] i.e.,

[6] If costs and revenues are accumulated to the end of the period instead of discounted to the beginning, both sides of (7) and (8) will be multiplied by the constant factor e^{rt_1} which will not affect the result.

[7] This can be shown as follows: suppose S and R have the optimum shapes that are economically possible. Then suppose $S+\lambda\sigma$ and $R+\lambda\rho$ to be all pairs of functions which would be preferable to S and R, but which are ruled out as economically impossible. It follows from (9) that

$$\int_{t_0}^{t}[(S + \lambda\sigma) - (R + \lambda\rho)]dt > 0, \qquad t_1 > t > t_0,$$

that is,

$$\int_{t_0}^{t}(\sigma - \rho)\lambda dt > \int_{t_0}^{t}(R - S)dt.$$

Since this must hold where $\lambda \to 0$, and since by condition (9) $\int_{t_0}^{t}(R-S)dt \not< 0$, it follows that $S \equiv R$.

$$S \equiv R.$$

Using the same methods as before, the solution now becomes

$$P + S\frac{\partial P}{\partial S} = C + S\frac{\partial C}{\partial S}. \tag{10}$$

That is, marginal costs = marginal revenue at every point of time, without further restriction.

III

We shall now turn to consider a particular example. Let

$$P = (50 - S)e^{0.1t},$$

$$C = (0.1R + 1 + 10/R)e^{0.15t},$$

$$r = 0.03,$$

$$t - t_0 = 2 \text{ years}.$$

The values of P and C imply that for given volumes of sales and production, prices and costs are increasing annually by about 11 per cent and 16 per cent respectively. Substituting in (7) and (8) and solving for S and R:

$$S = -0.5\Gamma e^{-0.07t} + 25, \tag{11}$$

$$R = 5\Gamma e^{-0.12t} - 5. \tag{12}$$

Applying these results to (2):

$$\int_0^2 (-0.5\Gamma e^{-0.07t} - 5\Gamma e^{-0.12t} + 20)dt = \quad .$$

Integrating and solving for Γ:

$$\Gamma = 6.11.$$

Substituting this value of Γ in (11) and (12):

$$S = -3.05e^{-0.07t} + 25, \tag{13}$$

$$R = 30.54e^{-0.12t} - 5. \tag{14}$$

For total production and total sales over the two-year period, we have:

$$\int_0^2 S dt = \int_0^2 R dt = 44.3.$$

It is of interest to compare these results with those achieved where marginal costs = marginal revenue at every point of time. Substituting for P and C in (10) and taking $S \equiv R$, we have:

$$(50 - 2S)e^{0.1t} - (0.2S + 1)e^{0.15t} = 0.$$

Whence

$$S = \frac{25 - 0.5e^{0.05t}}{1 + 0.1e^{0.05t}}. \tag{15}$$

By graphical integration of (6), we have approximately

$$\int_0^2 S dt = 44.3.$$

The approximate equality of the results in both cases is, of course, a coincidence.

The following table traces the history of (13), (14), and (15) over the two-year period at half-yearly intervals:

ANNUAL RATES OF PRODUCTION AND SALES

At t	"Inventory" Solution		"Hand-to-Mouth" Solution
	Sales (S)	Production (R)	$S \equiv R$
0	21.95	25.54	22.28
$\frac{1}{2}$	22.06	23.72	22.21
1	22.16	21.05	22.14
$\frac{3}{2}$	22.25	20.48	22.07
2	22.35	18.99	22.01

University of Michigan
Ann Arbor, Michigan

A NEW FORMULA FOR THE INDEX OF COST OF LIVING

By A. WALD

I SHALL GIVE in this paper a new formulation of the index formula I developed in an earlier paper,[1] and shall express it as a function of the income. Further I shall discuss some properties of this index formula and the criteria for its applicability.

I. THE NEW FORMULA

I shall first summarize the classical formula needed in our analysis. A point q in the n-dimensional Euclidean space with the co-ordinates $q^1, \cdots, q^n$ represents the quantities $q^1, \cdots, q^n$ of the goods nos. $1, \cdots, n$. Each individual has a scale of preferences in this space of goods. The scale of preferences can be described by an *indicator*, that is, a real function $I(q)$ such that $I(q) = I(\bar{q})$ if q and $\bar{q}$ are two indifferent sets of goods and $I(q) > I(\bar{q})$ if q is preferred to $\bar{q}$. If $I(q)$ is an indicator then every monotonically increasing function $F(I)$ of I is also an indicator. The set of points q for which the indicator has constant value is an indifference surface.

The quantities consumed are in general uniquely determined as functions of the income and the prices by the well-known optimum conditions

$$(*) \qquad \frac{\dfrac{\partial I}{\partial q^1}}{p^1} = \frac{\dfrac{\partial I}{\partial q^2}}{p^2} = \cdots = \frac{\dfrac{\partial I}{\partial q^n}}{p^n},$$

$$\sum_{i=1}^{n} p^i q^i = E,$$

where p^i are the prices. These conditions express the fact that among all sets of goods which can be purchased by the income E,[2] the individual chooses the set of goods for which the indicator has the greatest value.

Solving the above equations we get the quantities purchased expressed as functions of the prices and of income. If we consider a single time period with constant prices, then the quantities purchased will depend only on the income E, that is to say

$$q^1 = f^1(E),\ q^2 = f^2(E),\ \cdots,\ q^n = f^n(E).$$

These equations represent a curve in the space of goods which is

[1] "Zur Theorie der Preisindexziffern," *Zeitschrift für Nationalökonomie*, Wien, 1937.

[2] We understand by income the total expenditure for consumers' goods.

called an Engel curve. To each system of prices belongs a certain Engel curve.

The Engel curves can be determined empirically in each time period by observing the consumption of individuals belonging to different income classes. The interpretation of the curves thus obtained as Engel curves in the above sense is correct only if we can suppose that the individuals considered have essentially the same scale of preferences. This condition, of course, can be fulfilled only approximately.

The problem of the index of cost of living can be formulated as follows: Let us consider two time periods 0 and 1, denote by $p_0', \cdots, p_0^n$ the prices in the period 0, and by $p_1', \cdots, p_1^n$ the prices in the period 1. We suppose that we know the Engel curves C_0 and C_1 in the time period 0 and 1. We have to calculate for each income E_0 of the period 0 the *equivalent* income E_1 in the period 1. We say that the income E_1 in the period 1 is equivalent to the income E_0 in the period 0, if E_1 is the smallest income which enables us to buy with the prices of the period 1 a set of goods lying on the same indifference surface as the set of goods bought by E_0 in the period 0. The ratio E_1/E_0 between equivalent incomes is called the index of cost of living of the period 1 relative to the period 0 as basis. Let us denote this index by $P_{01}(E_0)$. The index depends in general on the income E_0.

I shall next indicate the derivation of my formula. Denote by $q_0=(q_0^1, q_0^2, \cdots, q_0^n)$ a point of the Engel curve C_0 and determine two points q_1 and $\hat{q}_1$ on the Engel curve C_1 such that

$$(1)\quad \sum p_0(q_1 - q_0) = \sum p_1(\hat{q}_1 - q_0) = 0 \text{ (summation over all goods).}$$

Such points q_1 and $\hat{q}_1$ certainly exist. If it should happen that $q_1=\hat{q}_1$, then by (1)

$$\sum p_0 q_1 = \sum p_0 q_0 \qquad \text{and} \qquad \sum p_1 q_1 = \sum p_1 q_0.$$

From the first of these equations follows $I(q_0) \geqq I(q_1)$ and from the second $I(q_0) \leqq I(q_1)$, where $I(q)$ denotes an indicator. Hence $I(q_0)=I(q_1)$ and $P_{01}(E_0)=\sum p_1 q_1 / \sum p_0 q_0$. The problem of constructing an index number could then be solved exactly using only the empirical data p_0, p_1, q_1. Consider therefore the case $q_1 \neq \hat{q}_1$.

The points q_1 and $\hat{q}_1$ will be near to the point q_0 if the two Engel curves C_0 and C_1 are not very far from each other. C_0 and C_1 can be distant from each other only if the price ratios in the period 0 are very different from the price ratios in the period 1. This will in general not be the case, if the time periods compared are not too distant.

Denote by $\hat{q}_0$ an arbitrary point on C_0 in the neighborhood of q_0 and such that

$$\sum p_1(\hat{q}_0 - q_0) \neq 0.$$

In order to be able to determine an approximation to the index of cost of living, we shall make the assumption:

(a) The indicator $I(q)$ can be approximated by a polynomial of the second degree in $q=(q^1, \cdots, q^n)$ in the tetrahedron T determined by the four points q_0, $\hat{q}_0$, q_1, $\hat{q}_1$.

If the tetrahedron T is small, the assumption (a) is justified.

If r_0 is a point on C_0 and r_1 a point on C_1 both lying in the tetrahedron T, then on account of the assumption (a), the following equation can easily be deduced:

$$
\begin{aligned}
I(r_1) - I(r_0) &= \frac{1}{2} \sum_{i=1}^{n} \left(\frac{\partial I(r_0)}{\partial q^i} + \frac{\partial I(r_1)}{\partial q^i} \right) (r_1{}^i - r_0{}^i) \\
&= \frac{1}{2} \sum (w_0 p_0 + w_1 p_1)(r_1 - r_0),
\end{aligned} \tag{2}
$$

where w_t denotes the marginal utility of money in r_t $(t=0, 1)$, that is, the value of the common ratio in the optimum condition.

If $\bar{r}_0$ and $\bar{\bar{r}}_0$ are *two* points on C_0, $\bar{r}_1$ and $\bar{\bar{r}}_1$ *two* points on C_1, all lying in the tetrahedron T we similarly get:

$$
I(\bar{r}_t) - I(\bar{\bar{r}}_t) = \frac{\bar{w}_t + \bar{\bar{w}}_t}{2} \sum p_t(\bar{r}_t - \bar{\bar{r}}_t) \qquad (t = 0, 1). \tag{3}
$$

From (2) and (3) we derive:

$$
\begin{aligned}
&\sum (w_1 p_1 + w_0 p_0)(q_0 - q_1) + (w_0 + \hat{w}_0) \sum p_0(\hat{q}_0 - q_0) \\
&\quad + \sum (w_1 p_1 + \hat{w}_0 p_0)(q_1 - \hat{q}_0) \\
&= 2\{ [I(q_0) - I(q_1)] + [I(\hat{q}_0) - I(q_0)] + [I(q_1) - I(\hat{q}_0)] \} = 0,
\end{aligned} \tag{4}
$$

and

$$
\begin{aligned}
&\sum (w_0 p_0 + w_1 p_1)(q_1 - q_0) + (w_1 + \hat{w}_1) \sum p_1(\hat{q}_1 - q_1) \\
&\quad + \sum (w_0 p_0 + \hat{w}_1 p_1)(q_0 - \hat{q}_1) \\
&= 2\{ [I(q_1) - I(q_0)] + [I(\hat{q}_1) - I(q_1)] + [I(q_0) - I(\hat{q}_1)] \} = 0,
\end{aligned} \tag{5}
$$

where w_t denotes the marginal utility of money in q_t and $\hat{w}_t$ in $\hat{q}_t$ $(t=0, 1)$.

From (1) and (4) we get:

$$
\frac{w_1}{w_0} = \frac{\sum p_0(\hat{q}_0 - q_0)}{\sum p_1(\hat{q}_0 - q_0)}; \tag{6}
$$

and from (1) and (5) we get

$$
\frac{\hat{w}_1}{w_0} = \frac{\sum p_0(\hat{q}_1 - q_1)}{\sum p_1(\hat{q}_1 - q_1)}. \tag{7}
$$

Let us denote the right-hand side of (6) by λ and the right-hand side of (7) by μ.

It can be easily shown: The Engel curves C_0 and C_1 and the marginal utility of money are linear functions of the income in the tetrahedron T.

The Engel functions $q_t^1 = f_t^1(E_t), \cdots, q_t^n = f_t^n(E_t)$ and the marginal utility of money $w_t(E_t)$ are the solutions of the $n+1$ equations,

$$\frac{\partial I}{\partial q_t^1} = w_t p_t^1 = \cdots = \frac{\partial I}{\partial q_t^n} = w_t p_t^n, \qquad \sum p_t q_t = E_t,$$

in the unknown quantities $q_t^1, \cdots, q_t^n, w_t$. Since $I(q)$ is a polynominal of the second order in the tetrahedron T, $\partial I/\partial q^i$ is linear in T, from which our proposition easily follows.

If r_1' and r_1'' are two points on C_1 lying in the tetrahedron T, then all points of the segment determined by r_1' and r_1'' are also points of C_1. For instance all points of the segment determined by q_1 and $\hat{q}_1$ are points of C_1.

From (1) we can easily derive:

$$I(q_1) \leqq I(q_0) \leqq I(\hat{q}_1).$$

Since $I(q)$ is continuous, there exists a point $\bar{q}_1$ which lies on the segment $q_1\hat{q}_1$ and for which

$$I(\bar{q}_1) = I(q_0). \tag{8}$$

Let us denote $\sum p_1 q_1$ by E_1, $\sum p_1 \hat{q}_1$ by $\hat{E}_1$, and $\sum p_1 \bar{q}_1$ by $\bar{E}_1$. It is obvious that the marginal utility of money $w_1(E)$ is a linear function of the income E in the interval $J = [E_1, \hat{E}_1]$. Hence we have in the interval J

$$\frac{w_1(E)}{w_0} = \alpha E + \beta, \tag{9}$$

where α and β are certain constants. We can calculate the values of α and β, because the value $w_1(E)/w_0$ is already known for $E = E_1$ and $E = \hat{E}_1$. In fact it is

$$\frac{w_1(E_1)}{w_0} = \frac{w_1}{w_0} = \lambda \qquad \text{and} \qquad \frac{w_1(\hat{E}_1)}{w_0} = \frac{\hat{w}_1}{w_0} = \mu,$$

where λ means the right-hand side of the equation (6) and μ the right-hand side of (7).

For α and β we get the values:

$$\alpha = \frac{\mu - \lambda}{\hat{E}_1 - E_1} \qquad \text{and,} \qquad \beta = \lambda - E_1 \frac{\mu - \lambda}{\hat{E}_1 - E_1}. \tag{10}$$

Denoting the marginal utility of money in the point $\bar{q}_1$ by $\bar{w}_1$, we get

$$\alpha E_1 + \beta = \frac{w_1}{w_0} = \lambda,$$

$$\alpha \hat{E}_1 + \beta = \frac{\hat{w}_1}{w_0} = \mu, \tag{11}$$

$$\alpha \bar{E}_1 + \beta = \frac{\bar{w}_1}{w_0}.$$

On account of (2), (3), and (8) we have

$$\sum (w_0 p_0 + w_1 p_1)(q_1 - q_0) + (w_1 + \bar{w}_1) \overline{\sum} p_1(\bar{q}_1 - q_1) = 0,$$

and therefore on account of (1)

$$\sum p_1(q_1 - q_0) + \left(1 + \frac{\bar{w}_1}{w_1}\right) \sum p_1(\bar{q}_1 - q_1) = 0. \tag{12}$$

From (11) follows

$$\frac{\bar{w}_1}{w_1} = \frac{\alpha \bar{E}_1 + \beta}{\alpha E_1 + \beta}. \tag{13}$$

If we substitute in (12) for $\bar{w}_1/w_1$ the right-hand side of (13), we get

$$\sum p_1(q_1 - q_0) + \left(1 + \frac{\alpha \bar{E}_1 + \beta}{\alpha E_1 + \beta}\right)(\bar{E}_1 - E_1) = 0.$$

This is an equation of the second degree in $\bar{E}_1$. The roots of this equation are

$$\bar{E}_1 = \frac{-\beta + \epsilon \sqrt{(\alpha E_1 + \beta)^2 - \alpha D}}{\alpha}, \tag{14}$$

where

$$D = (\alpha E_1 + \beta) \sum p_1(q_1 - q_0) \quad \text{and} \quad \epsilon = +1 \text{ or } -1.$$

From (14) we get

$$\frac{\bar{w}_1}{w_0} = \alpha \bar{E}_1 + \beta = \epsilon \sqrt{(\alpha E_1 + \beta)^2 - \alpha D}.$$

Since $\bar{w}_1/w_0$ is a nonnegative number, ϵ must be equal to $+1$. Therefore we have

$$\frac{\bar{w}_1}{w_0} = + \sqrt{(\alpha E_1 + \beta)^2 - \alpha D},$$

and hence

$$\frac{\bar{w}_1}{w_1} = \sqrt{\frac{(\alpha E_1 + \beta)^2 - \alpha D}{(\alpha E_1 + \beta)^2}} = \sqrt{1 - \frac{\alpha D}{(\alpha E_1 + \beta)^2}}$$

$$= \sqrt{1 - \frac{\alpha \sum p_1(q_1 - q_0)}{\alpha E_1 + \beta}}.$$

Since, according to (1), $\sum p_1 q_0 = \sum p_1 \hat{q}_1 = \hat{E}_1$, we get on account of (10) and (11),

$$\frac{\bar{w}_1}{w_1} = \sqrt{1 - \frac{\alpha(E_1 - \hat{E}_1)}{\lambda}} = \sqrt{1 - \frac{\lambda - \mu}{\lambda}} = \sqrt{\frac{\mu}{\lambda}}. \tag{15}$$

If we substitute in (12) for $\bar{w}_1/w_1$ the value $\sqrt{\mu/\lambda}$, then we get from (12) on account of (1)

$$P_{01}(E_0) = \frac{E_1}{E_0} = \frac{\sum p_1 \left(q_0 + \sqrt{\frac{\mu}{\lambda}} q_1 \right)}{\sum p_0 \left(q_0 + \sqrt{\frac{\mu}{\lambda}} q_1 \right)}. \tag{16}$$

(16) is the same formula which I demonstrated in my paper already mentioned. It can be transformed into a form which for certain purposes is more convenient.

Suppose the Engel curves C_0 and C_1 are given by the following equations:

The equations of C_0: $q_0^i = \alpha_0^i E + \beta_0^i$, The equations of C_1: $q_1^i = \alpha_1^i E + \beta_1^i$ $(i = 1, 2, \cdots, n)$.

We introduce the notations:

$$\sum \alpha_i p_j = a_{ij} \quad \text{and} \quad \sum \beta_i p_j = b_{ij} \quad \text{(summation over all goods)}.$$

It is obvious that $a_{11} = a_{00} = 1$, and $b_{11} = b_{00} = 0$.
We get from (6) and (7)

$$\lambda = \frac{a_{00}}{a_{01}} = \frac{1}{a_{01}} \quad \text{and} \quad \mu = \frac{a_{10}}{a_{11}} = a_{10}. \tag{17}$$

On account of (1) we have

$$\sum p_0 q_0 = \sum p_0 q_1 = E_0. \tag{18}$$

Since $\sum p_1 q_1$ is equal to E_1 and therefore

$$q_1' = \alpha_1' E_1 + \beta_1', \cdots, q_1^n = \alpha_1^n E_1 + \beta_1^n,$$

we get on account of (1)

$$a_{10}E_1 + b_{10} = E_0.$$

Hence

$$E_1 = \frac{E_0 - b_{10}}{a_{10}} = \sum p_1 q_1. \tag{19}$$

It is obvious that

$$\sum p_1 q_0 = a_{01}E_0 + b_{01}. \tag{20}$$

If we substitute in (16) for λ, μ, $\sum p_1 q_0$, $\sum p_1 q_1$, $\sum p_0 q_0$, and $\sum p_0 q_1$, their expressions from (17), (18), (19), and (20), then we get

$$P_{01}(E_0) = \frac{a_{01} + \frac{1}{a_{10}}\sqrt{a_{10}a_{01}}}{1 + \sqrt{a_{01}a_{10}}} + \frac{1}{E_0} \cdot \frac{b_{01} - \frac{b_{10}}{a_{10}}\sqrt{a_{01}a_{10}}}{1 + \sqrt{a_{01}a_{10}}}, \tag{21}$$

or

$$P_{01}(E_0) = \sqrt{\frac{a_{01}}{a_{10}}} + \frac{1}{E_0} \cdot \frac{b_{01} - \frac{b_{10}}{a_{10}}\sqrt{a_{01}a_{10}}}{1 + \sqrt{a_{01}a_{10}}} \tag{22}$$

This is the formula sought. If the α and β coefficients for the Engel curves are statistically determined all the magnitudes in (22) are known.

The assumption that the indicator is a polynomial may be represented geometrically as follows: For the sake of simplification we shall suppose that we have only two consumers' goods Nos. 1 and 2. The two axes in Figure 1 represent the quantities of these goods. C_0 and C_1 denote the Engel curves in the periods 0 and 1 respectively. A straight line given by the equation $p_0^1 q^1 + p_0^2 q^2 = E$ is called a 0-budget line. Obviously all 0-budget lines—for different E—are parallel. Similarly a 1-budget line is defined by $p_1^1 q^1 + p_1^2 q^2 = E$. Denote by E_0 the income in the period 0 for which we want to calculate the index $P_{01}(E_0)$, and by q_0 the point of C_0 purchased by the amount E_0. We shall determine the points q_1 and $\hat{q}_1$ of C_1. The point q_1 satisfies the equation $\sum p_0(q_1 - q_0)$. Hence q_1 is the intersection point of the Engel curve C_1 with the 0-budget line which goes through q_0. The point $\hat{q}_1$ satisfies the equation $\sum p_1(\hat{q}_1 - q_0) = 0$. Hence $\hat{q}_1$ is the intersection point of the Engel curve C_1 with the 1-budget line which goes through q_0.

Our assumption means that the indicator $I(q)$ can be well approximated by a polynomial of the second degree in the interior of the shaded triangle determined by the points q_0, q_1, $\hat{q}_1$.[3]

[3] Originally we have supposed that $I(q)$ is a polynomial of the second degree in the tetrahedron determined by the points q_0, $\hat{q}_0$, q_1, $\hat{q}_1$. Since $\hat{q}_0$ can be chosen as near to q_0 as we like, it suffices to assume that in the triangle considered.

Is it possible to give statistical criteria for the validity of the polynomial assumption? This is indeed possible to a certain extent. The assumption that $I(q)$ is a polynomial of the second degree in the triangle q_0, q_1, $\hat{q}_1$ implies the linearity of the Engel curve C_1 between q_1 and $\hat{q}_1$. But the inverse is not true in general. For *practical* purposes the inverse conclusion is, however, rather plausible. That is, if we see that the Engel curve C_1 can be well approximated by a straight line be-

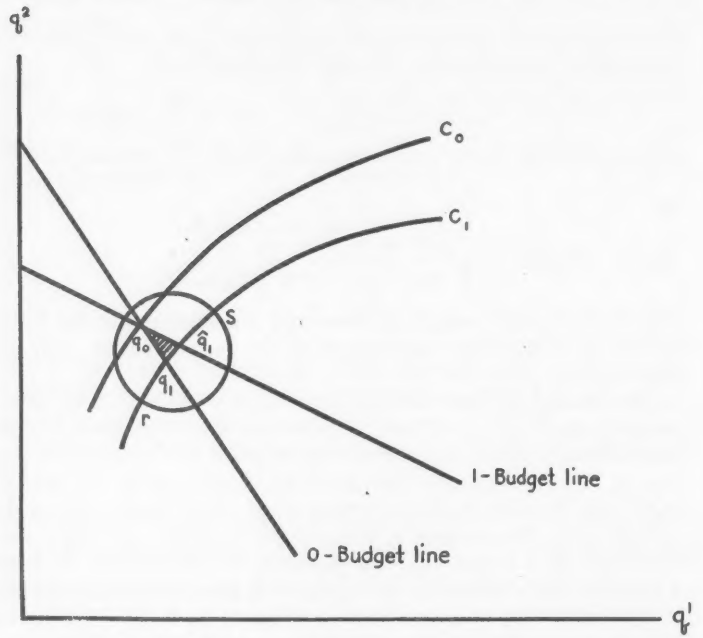

FIGURE 1.

tween two points r and s such that the circle with the diameter rs contains in its interior the triangle q_0, q_1, $\hat{q}_1$, we can regard the assumption of a polynomial indicator as justified. Such a situation is suggested in Figure 1.

It should be mentioned that R. G. D. Allen and A. L. Bowley[4] assume the linearity of the Engel curves and they found this assumption verified by extensive empirical investigations of family-budget data. The income interval in which the Engel curves were found to be linear

[4] *Family Expenditure*, P. S. King and Sons, London, 1935.

is much greater than is needed for the verification of the assumption of a polynomial indicator.

II. THE INDEX OF COST OF LIVING AS A FUNCTION OF THE INCOME

From formula (22) follows that the index as a function of the income is a rectangular hyperbola. Let us denote

$$\sqrt{\frac{a_{01}}{a_{10}}} \quad \text{by} \quad C_{01} \quad \text{and} \quad \frac{b_{01} - \frac{b_{10}}{a_{10}}\sqrt{a_{01}a_{10}}}{1 + \sqrt{a_{01}a_{10}}}$$

by d_{01}; then we get

$$P_{01}(E_0) = C_{01} + \frac{d_{01}}{E_0}.$$

The value C_{01} is always positive. The value d_{01} can be positive or negative. This depends on the fact whether the luxury goods became in the period 1 relatively more expensive than the necessities, or inversely.

It should be mentioned that Allen and Bowley show in their book, *Family Expenditure*, p. 16, that the index is a rectangular hyperbola, if (1) we suppose the linearity of the Engel curves, and (2) we use as index formula $P_{01}(E)$, Laspeyres' formula, that is to say,

$$P_{01}(E_0) = \frac{\sum p_1 q_0}{\sum p_0 q_0}.$$

The hyperbola derived by means of formula (22) is different from the hyperbola established by Allen and Bowley, because formula (22) is different from Laspeyres' formula.

III. THE INDEX FORMULA (22) AND THE LIMITS GIVEN BY STAEHLE

Combining the limits obtained by Haberler[5] and Konüs,[6] Staehle[7] arrives at the limits:

$$(23) \qquad \frac{\sum p_1 q_1}{\sum p_0 q_1} \leqq P_{01}(E_0) \leqq \frac{\sum p_1 q_0}{\sum p_0 q_0},$$

[5] *Der Sinn der Indezahlen*, Tubingen, 1927.

[6] See for instance Ragner Frisch, "Annual Survey of General Economic Theory: The Problem of Index Numbers," ECONOMETRICA, Vol. 4, January, 1936, pp. 1–38.

[7] "A Development of the Economic Theory of Price Index Numbers," *The Review of Economic Studies*, 1935.

where q_1 denotes the point of the Engel curve C_1 for which

$$\sum p_0 q_1 = \sum p_0 q_0.$$

The value of $P_{01}(E_0)$ given by our formula (22) lies between the two limits of Staehle. Indeed, the points q_0 and q_1 in (16) are the same points as the points q_0 and q_1 in the limits of Staehle.[8] Since the expression $\sqrt{\mu/\lambda}$ in (16) is nonnegative, and (16) is a monotonic function of $\sqrt{\mu/\lambda}$, the value of (16) must lie between

$$\frac{\sum p_1 q_0}{\sum p_0 q_0} \quad \text{and} \quad \frac{\sum p_1 q_1}{\sum p_0 q_1},$$

that is to say, between the two limits of Staehle.

IV. THE INDEX FORMULA (22) AND THE DOUBLE-EXPENDITURE METHOD OF RAGNAR FRISCH

The double-expenditure method has been developed by Ragnar Frisch in his papers, "The Problem of Index Numbers," ECONOMETRICA, Vol. 4, January, 1936, p. 29, and "The Double-Expenditure Method," ECONOMETRICA, Vol. 6, January, 1938, p. 85. The method can be described as follows: Let us denote by q_0 a point of the Engel curve C_0 and by q_1 a point of the Engel curve C_1. If the indicator $I(q)$ can be approximated by a polynomial of the second degree, then we have

$$\begin{aligned} I(q_1) - I(q_0) &= \tfrac{1}{2}\sum (w_0 p_0 + w_1 p_1)(q_1 - q_0) \\ &= \tfrac{1}{2}(w_0 \sum p_0 q_1 - w_1 \sum p_1 q_0) \\ &\quad + \tfrac{1}{2}(w_1 \sum p_1 q_1 - w_0 \sum p_0 q_0), \end{aligned}$$

where $w_t (t=0, 1)$ denotes the marginal utility of money in the point q_t. Hence for a point q_1, for which $I(q_1) = I(q_0)$, the following equation will hold true:

$$(24) \qquad (w_0 \sum p_0 q_1 - w_1 \sum p_1 q_0) + (w_1 \sum p_1 q_1 - w_0 \sum p_0 q_0) = 0.$$

Frisch makes the assumption that, if $I(q_1) = I(q_0)$, then the relation

$$(25) \qquad \frac{w_1}{w_0} = \frac{p_0 q_0}{p_1 q_1}$$

is valid approximately. From (24) and (25) follows that, if $I(q_1) = I(q_0)$, then

$$\sum p_1 q_1 \sum p_0 q_1 = \sum p_1 q_0 \sum p_0 q_0.$$

[8] Also in formula (16) q_1 denotes the point of the Engel curve C_1, for which $\sum p_0(q_1 - q_0) = 0$.

If a point q_0 of C_0 is given, we determine the point q_1 of C_1 for which $I(q_1)=I(q_0)$, by choosing that point q_1 of C_1 for which the equation holds true:

$$\sum p_1q_1 \sum p_0q_1 = \sum p_1q_0 \sum p_0q_0.$$

The relation between this method and ours consists in the fact that Frisch also approximates the indicator by a polynomial of the second degree. Besides this Frisch makes the additional assumption that the relation (25) is fulfilled. It should be mentioned that if we assume that $I(q)$ can be approximated by a polynomial of the second degree, it is superfluous to make additional assumptions, because the polynomial assumption already suffices for the unique determination of the index. If the assumption (25) is fulfilled, the double-expenditure method will surely give very good results. However, the validity of (25) cannot be assumed generally. In all cases where the approximation by a polynomial of the second degree is justified, formula (22) gives the best approximation to the true value of the index. In cases where the approximation by a polynomial of the second degree cannot be assumed, we cannot say anything definite about the bias in the results given by the formula (22) or by the double-expenditure method.[9,10]

We shall show that also the index given by the double-expenditure method lies between the limits of Staehle, if the following assumption is fulfilled:

(A) If $\bar{q}_1$ and $\bar{\bar{q}}_1$ are two points of C_1 for which $\sum p_1\bar{\bar{q}}_1 \geqq \sum p_1\bar{q}_1$ then also $\sum p_0\bar{\bar{q}}_1 \geqq \sum p_0\bar{q}_1$. This assumption will be fulfilled in most practical cases. But it is not necessarily always fulfilled. Even though $\sum p_1\bar{\bar{q}}_1 \geqq \sum p_1\bar{q}_1$, it may happen that, for some goods, $\bar{\bar{q}}_1{}^i < \bar{q}_1{}^i$.[11] If for these goods $p_0{}^i/p_1{}^i$ is much greater than for the other goods, we may get $\sum p_0\bar{\bar{q}}_1 < \sum p_0\bar{q}_1$.

Let q_0, q_1, and $\hat{q}_1$ have the same meaning as before. In order to demonstrate that the index given by the double-expenditure method

[9] We can only say that the bias in the formula (22) cannot be greater than the difference between the two limits of Staehle. This follows from the fact that the index given by (22) lies always between the two limits of Staehle.

[10] Dr. Wald's remark must not be interpreted to mean that the double-expenditure method can at best give an approximation nearly as good as Dr. Wald's method but never a better approximation. My additional "superfluous" assumption may indeed in many cases correct for part of the error committed by assuming the indicator as a polynomial. This seems obvious a priori and has been verified by calculations on constructed examples. The double-expenditure method turned out as the more accurate of the two about as frequently as Dr. Wald's method did. The deviations from the correct values were of the same order of magnitude. Both methods gave very good results but Dr. Wald's method proved to be much more laborious.—RAGNAR FRISCH.

[11] See, for instance, Allen and Bowley, *Family Expenditure*, pp. 13 and 134.

lies between the limits of Staehle, we have only to show that $\sum p_0q_0\sum p_1q_0$ lies between the values $\sum p_0q_1\sum p_1q_1$ and $\sum p_0\hat{q}_1\sum p_1\hat{q}_1$. We consider two cases:

(I) $\sum p_1q_1 \leqq \sum p_1\hat{q}_1$. We shall show that in this case

$$\sum p_0q_1 \sum p_1q_1 \leqq \sum p_0q_0 \sum p_1q_0 \leqq \sum p_0\hat{q}_1 \sum p_1\hat{q}_1.$$

Since $\sum p_0q_1 = \sum p_0q_0$ and $\sum p_1\hat{q}_1 = \sum p_1q_0$ we have only to demonstrate that

$$\begin{cases} \sum p_1q_1 \leqq \sum p_1q_0 = \sum p_1\hat{q}_1, \\ \sum p_0\hat{q}_1 \geqq \sum p_0q_0 = \sum p_0q_1. \end{cases} \tag{26}$$

The first of the inequalities of (26) is fulfilled on account of the assumption (I) and the second follows from our assumption (A).

(II) $\sum p_1q_1 \geqq \sum p_1\hat{q}_1$. We shall show that in this case

$$\sum p_0q_1 \sum p_1q_1 \geqq \sum p_0q_0 \sum p_1q_0 \geqq \sum p_0\hat{q}_1 \sum p_1\hat{q}_1.$$

Since $\sum p_0q_1 = \sum p_0q_0$ and $\sum p_1\hat{q}_1 = \sum p_1q_0$ we have only to demonstrate that

$$\begin{cases} \sum p_1q_1 \geqq \sum p_1q_0 = \sum p_1\hat{q}_1, \\ \sum p_0\hat{q}_1 \leqq \sum p_0q_0 = \sum p_0q_1. \end{cases} \tag{27}$$

The first of the inequalities (27) is fulfilled on account of (II), and the second follows from the assumption (A).

If the assumption (A) is not fulfilled for the points q_1 and $\hat{q}_1$ then the index calulated by the double-expenditure method will in general fall outside the limits of Staehle.

Finally we want to calculate the index formula of the double-expenditure method under the assumption that the Engel curves C_0 and C_1 are linear. With the same notation as before we have:

$$\sum p_0q_0 = E_0; \qquad \sum p_0q_1 = \sum \alpha_1p_0E_1 + \sum \beta_1p_0 = a_{10}E_1 + b_{10};$$
$$\sum p_1q_0 = \sum \alpha_0p_1E_0 + \sum \beta_0p_1 = a_{01}E_0 + b_{01}; \qquad \sum p_1q_1 = E_1.$$

According to the double-expenditure method, the point q_1, which is equivalent to q_0, is given by the equation

$$\sum p_0q_0 \sum p_1q_0 = \sum p_0q_1 \sum p_1q_1.$$

If we substitute for $\sum p_0q_0, \sum p_0q_1, \sum p_1q_0, \sum p_1q_1$ the above expressions, we get

$$E_0(a_{01}E_0 + b_{01}) = E_1(a_{10}E_1 + b_{10}).$$

Hence

$$E_1 = \frac{-b_{10} + \sqrt{b_{10}^2 + 4a_{10}a_{01}E_0^2 + 4a_{10}b_{01}E_0}}{2a_{10}},$$

and

$$(28) \qquad P_{01}(E_0) = \frac{-b_{10} + \sqrt{b_{10}^2 + 4a_{10}a_{01}E_0^2 + 4a_{10}b_{01}E_0}}{2a_{10}E_0}.$$

This index formula becomes equivalent to formula (22) if

$$(29) \qquad (a_{10}b_{01})^2 = a_{10}a_{01}(b_{10})^2.$$

We get in fact from (28) on account of (29)

$$P_{01}(E_0) = \frac{-b_{10} + \sqrt{(2E_0\sqrt{a_{10}+a_{01}} + b_{10})^2}}{2a_{10}E_0} = \sqrt{\frac{a_{01}}{a_{10}}},$$

and the formula (22), as can be easily shown, become also equivalent to the formula

$$P_{01}(E_0) = \sqrt{\frac{a_{01}}{a_{10}}}.$$

If the Engel curves C_0 and C_1 are straight lines through the origin, then $b_{01}=b_{10}=0$ and (29) is fulfilled. We see further that in this case

$$a_{01} = \frac{\sum p_1q_0}{\sum p_0q_0} \quad \text{and} \quad \frac{1}{a_{10}} = \frac{\sum p_1q_1}{\sum p_0q_1}.$$

Hence the index formulae (28) and (22) give the same result:

$$P_{01}(E_0) = \sqrt{\frac{a_{01}}{a_{10}}} = \sqrt{\frac{\sum p_1q_0}{\sum p_0q_0} \cdot \frac{\sum p_1q_1}{\sum p_0q_1}},$$

that is to say, the index formula is reduced to the ideal formula of Irving Fisher.[12]

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[12] Ragnar Frisch (*loc. cit.*) mentions this fact for his double-expenditure method.

ON COMBINING MARKET AND BUDGET DATA IN DEMAND STUDIES: A SUGGESTION

By J. Marschak

In the work done to obtain demand functions from market data, the influence of income changes on demand has been usually eliminated (by Professor Schultz and others) by regarding the national income per head, R, as a linear component of the demand function; sometimes a parameter supposed to characterize the distribution (such as Pareto's α or some index of concentration)[1] was used as another such component, e.g.:

$$X = ap + bR + c\alpha + d, \tag{1}$$

where a, b, c, d are constants and p the price of the commodity (prices of other commodities may or may not be included as further variables). X, the quantity consumed by the community, is taken from sales data, or from production statistics corrected for export surpluses and changes of stocks.

This material can, however, be supplemented, and the assumed functional relationship between demand and income given an economically more meaningful form by the use of budget (or Engel) functions. Let the family's consumption x be represented as an unknown function of price p and income r ("personal demand function"):[2]

$$x = x(r, p).$$

From budget data collected when the price had a known value p', the personal budget function $x = x(r, p')$ is obtained. On the other hand, independent data on prices (p_i) and on national consumption (X_i) are available for a number of time-points i ($i = 1, 2, \cdots$); and, for the same time points, the income-distribution functions $f_i(r)$ are given, [where $\int_0^\infty f_i(r)dr = 1$]. Then

$$X_i = \int_0^\infty x(r, p_i)f(r)dr \tag{2}$$

is a function of price only. Example: assume a linear budget function

$$x = \alpha + \beta r,$$

[1] H. Staehle, *Die Analyse von Nachfragekurven*, Bonn, 1929, and his recent articles in the *Review of Economic Statistics*, 1937 and 1938.

[2] Here and elsewhere in this article, allowance might be made for time lags between the variables involved. In what follows, instead of saying "time point i," we might say, more generally "observation set i." Time lags have only been neglected here to simplify the exposition.

each of the two parameters depending linearly on the prevailing price p:

$$\alpha = a + kp,$$
$$\beta = b + lp;$$

hence

$$x = a + kp + br + lpr.^3 \tag{3}$$

From budgets collected when $p = p'$ the parameters α' and β' of the function

$$x = \alpha' + \beta' r$$

are obtained. The equations

$$\begin{aligned} \alpha' &= a + kp', \\ \beta' &= b + lp', \end{aligned} \tag{4}$$

hold, where a, b, k, l are the unknown parameters in (3). The national consumption (2) at time i is equal to

$$X_i = \int_0^\infty (a + kp_i + br + lp_i r) f(r)\,dr; \tag{5}$$

and

$$\int_0^\infty f_i(r)\,dr = 1, \tag{6}$$

$$\int_0^\infty r f_i(r)\,dr = R_i \equiv \text{national income per family.}$$

Hence

$$X_i = a + kp_i + bR_i + lp_i R_i; \tag{7}$$

substituting for a and b from (4) and collecting the known terms on the left-hand side:

$$X_i - \alpha' - \beta' R_i = k(p_i - p') + lR_i(p_i - p') \qquad (i = 1, 2, \cdots),$$

k, l[4] can be found from these equations by least squares, then a, b from (4). Both the "personal demand function" (3) and the corresponding "national demand function" (7) are thus found.

The field is also open for more general assumptions. For example if the assumed personal function is a quadratic in income and price

[3] On the less general assumption that the budget line merely shifts up and down with changing prices the term in pr would vanish.

[4] $l = 0$ if the narrower assumption of the preceding footnote is made; the coefficient of R is then simply β', and obtained directly from the budget study.

$$x(r, p) = a + kp + br + lpr + mp^2 + nr^2, \tag{8}$$

the corresponding budget function at a price $p = p'$, arranged according to the powers of r, is

$$x(r) = \alpha' + \beta' r + \gamma' r^2,$$

where

$$\begin{aligned} \alpha' &= a + kp' + mp'^2, \\ \beta' &= b + lp', \\ \gamma' &= n, \end{aligned} \tag{9}$$

and α', β', γ' are known.

From (2), (6), and (8)

$$X_i = a + kp_i + mp_i^2 + bR_i + lpR_i + n\mu_{2,i}, \tag{10}$$

where $\mu_{2,i}$ is the second moment (about zero) of the income distribution $f_i(r)$. Substituting from (9) for a, b, n, and collecting the known terms

$$\begin{aligned} X_i - (\alpha' + \beta' R_i + \gamma' \mu_{2,i}) &= k(p_i - p') + m(p_i^2 - p_i'^2) \\ &+ lR_i(p_i - p') + \cdots \qquad (i = 1, 2, \cdots). \end{aligned}$$

k, l, and m can be found from this set of observation equations, then a, b, n from (9).

It is seen that no "distribution law" (such as Pareto's) need be assumed; although if such a law—if only for a limited income range—is available, some calculations may be saved. Nor is it essential, although it is convenient, to make assumptions as to the form of the personal demand function $x(r, p)$.[5]

[5] It was seen above that the personal demand function (3), although itself not unduly simple, leads to a national demand function (7) where no other parameters of the income distribution besides average income appear. If the second power of income appears in the personal budget function—as in (8)—the second moment of the distribution appears in the national budget function; which means, incidentally, that Pareto's α—if his law is assumed—enters the equation not linearly—as in (1)—but in the form $1/(2-\alpha)$.

The personal budget functions assumed by R. Roy (*Metron*, 1930) are of particular interest because they map out certain extreme hypotheses. Adapting his notation to ours, his personal budget functions seem to be:

$$\left.\begin{aligned} x_a(r) &= \frac{r}{p_a} + \xi_a \quad \text{for } r < p_a\xi_a \\ x_a(r) &= \xi_a \qquad \text{for } r > p_a\xi_a \end{aligned}\right\} \text{for the aggregate of } \textit{necessaries } (a)$$

and

$$\left.\begin{aligned} x_b(r) &= 0 \qquad\qquad\quad \text{for } r < p_a\xi_a \\ x_b(r) &= \frac{r - p_a\xi_a}{p_b} \quad \text{for } p_a\xi_a < r < p_b\xi_b \\ x_b(r) &= \xi_b \qquad \text{for } r > p_b\xi_b \end{aligned}\right\} \text{for the aggregate of } \textit{luxuries } (b)$$

where, for each aggregate, the quantity index, the price index, and the saturation-

It is possible to proceed purely numerically. Calculate for each point of time (i) the national demand which would take place if the incomes changed as they actually did but the price were kept at the level (p') it had at the time of the budget study. This hypothetical demand may be written as

$$X_{i,p'} = \int_0^\infty x(r, p')f(r)dr$$

and can be found by summing the products: (consumption at income r as recorded in the budget study)$\times$(proportion of people having income r). We obtain then a time series of (1) the actual X_i, (2) the hypothetical $X_{i,p'}$, and (3) the prices, and can fit a function

$$X = g(X_{p'}, p), \tag{11}$$

from which, e.g., the elasticities of demand with respect to price can be calculated. The form of the function g will depend on the price and income distribution which happened to exist at the time of the budget study. This arbitrary standard (analogous to those implied in the technique of "standardized death-rates") is, however, no practical drawback. The disadvantage of (11) compared with, say (10) is its failure to show directly the way in which income changes affect national demand at a given price. It has, on the other hand, the advantage of not relying on any smoothing formulae.[6]

All Souls College
Oxford, England

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quantity index are denoted, respectively, by x_a, p_a, ξ_a, and x_b, p_b, ξ_b. Integrating as in (2), national demand functions for the two aggregates are obtained. They yield demand functions for single goods when further extreme hypotheses are tried for each aggregate: the single quantities within each aggregate vary in the same proportion; or, the expenditures on single commodities vary in the same proportion.—The two pairs of extreme hypotheses, and not (as is often thought) the assumption of Pareto's law, form the essential of Roy's contribution. The same is true of Wiśniewsky (ECONOMETRICA, 1935) whose assumptions for (single) luxury goods are, however, more general than Roy's. Pareto's (*Giornale degli Economisti*, 1895, p. 65) assumption for grain could be written $x = ms^b/(1+s^b)$ where $s = r/p$, and m, b are unknown constants. All three authors use (2) and some analytical distribution function $f(r)$, and Pareto uses also market (but not budget) data.

[6] General relationships between personal and collective demand functions are studied in more detail in a forthcoming article in the *Review of Economic Statistics*, November, 1939.

THE EFFECT OF THE UNDISTRIBUTED PROFITS TAX UPON THE DISTRIBUTION OF CORPORATE EARNINGS—A STATISTICAL APPRAISAL*

By Francis McIntyre

I

The aim of this paper is an appraisal of American experience with the taxation of undistributed profits during 1936 and 1937. It is in no sense an evaluation of the particular legislation involved, nor an attack on or defense of the idea of an undistributed profits tax. Our effort is rather the measurement, subject to the limitations of the available data, of the effect of the Revenue Act of 1936 upon corporate dividend policy.

Of course corporate dividend policy is an amazingly complex and heterogeneous economic institution. The decision to retain and reinvest earnings being complementary to the decision to pay out such earnings to stockholders in the form of dividends, corporate dividend policy may on some occasions be exclusively reinvestment policy, and on other occasions exclusively distribution policy. Probably it is most frequently a compromise between the two.

There is, obviously, no simple answer to the question, what proportion of earnings "ought" to be retained by a given company in a given year. Still less are generalizations for all industry possible.[1] We may, however, analyze the available statistical evidence in an effort to discover what have been the chief determinants of corporate investment and dividend policy, and this is attempted in Section III, below.

II

A tax on those profits of corporations which were not paid out to stockholders in the form of dividends is not at all new. American postcolonial experience with personal income taxation dates from 1861. This and other tax legislation of the Civil War period was regarded as of a purely emergency character, but the 1864 law contained the interesting provision that the earnings of corporations were taxable

* A portion of this paper was read before the Econometric Society at Detroit, December 28, 1938. For computational and other assistance in the preparation of this paper I am indebted to Mr. Forrest Danson, Miss Mary Jo Lawley, Miss Emma Manning, and Mrs. Vera von Bruening, all of the staff of the Cowles Commission, and Mr. Tynan Smith of the University of Chicago.

[1] For an extremely able presentation of the myriad factors determining corporate decisions to reinvest, see *The Investment Decisions of Industry*, by Shelby Cullom Davis of the Delaware Fund, Inc., read before the American Economic Association at Detroit, December 28, 1938, and privately printed.

as income to stockholders whether or not distributed as dividends. The period following the war saw restored to a new vigor adequate to support the Federal budget those sources of revenue then regarded as more normal and appropriate, and the modest income-tax rates in force were totally abolished in 1872. A short-lived revival of the personal income tax in 1894 was terminated by the decisions of the U.S. Supreme Court in *Pollock* vs. *The Farmers' Loan and Trust Co.*,[2] declaring personal income levies to be unapportioned direct taxation, and hence unconstitutional. The Congress of 1909 did not regard these decisions as prohibiting a tax on corporate income, however, and the Supreme Court later sustained this view.[3]

While legal opinion seems to regard the Flint case (which sustained corporate income taxes as a valid excise) as "better law" than the Pollock case, the legislatures of 36 states spared the Court any problem of review of its earlier decision by ratifying, between July, 1909 and February, 1913, the Sixteenth Amendment to the Constitution of the United States, providing for Federal taxation of incomes from whatever source derived. The first personal income tax under this amendment provided very modestly graduated rates, the maximum being 6 per cent on net incomes in excess of $500,000. Steeper graduation of rates followed almost immediately, however, and with this development came the first important efforts by individuals to avoid the higher rates to which large incomes subjected them, by diverting a portion of their incomes into corporations from which no dividends would be received. The personal holding company is, of course, simply a single case in point.[4]

The impression should not be given, however, that the elimination of this legal opportunity for tax avoidance was the only, or even the chief, purpose of the Undistributed Profits Tax provisions of the Revenue Act of 1936. In the early years (1913–16) of our modern income tax there were provisions for the taxation of unreasonable amounts of corporation profits withheld from stockholders. But these were insignificant in effect, and in the 1918 law did not appear. In 1934 the taxation of personal holding companies to discourage that means of tax

[2] 157 U.S. (1895) 429; and 158 U.S. (1895) 601.

[3] *Flint* vs. *Stone-Tracy Company*, 220 U.S. (1910) 107.

[4] Frederic C. Howe, *Taxation and Taxes in the United States under the Internal Revenue System, 1791–1895*, New York, Crowell and Co., 1896, pp. 90–97.

Davis R. Dewey, *Financial History of the United States*, 5th edition, New York, Longmans, Green and Co., 1915, pp. 305–306, 486–491.

William J. Schultz, *American Public Finance and Taxation*, revised edition, New York, Prentice-Hall, Inc., 1934, pp. 456–459, 525–528.

Alfred G. Buehler, *The Undistributed Profits Tax*, New York, McGraw-Hill, 1937, pp. 3–7.

avoidance was begun, and in 1936 a surtax was levied on the undistributed profits of corporations, whether or not such profits were withheld in order to avoid the higher brackets of individual income taxes. It must be remembered that the individual proprietorship or partnership was already paying such higher taxes. The undistributed profits of a partnership are regarded as, and taxed as, income to the individuals, whether plowed back into the business or not. At its worst or best, however desirable or undesirable one may believe it to be, the Undistributed Profits Tax merely equalized to some extent the positions of the corporation and the unincorporated enterprise.

During the House hearings on the new tax proposal, Commissioner of Internal Revenue Helvering advanced this point with the following constructed example:

> If four single men with no other taxable income form a partnership, which earns $1,000,000, the government receives $517,136. If these same men conduct their business as a corporation, and pay themselves salaries of $25,000 each, but no dividends, the government receives $145,656 in income taxes. Even if the corporation distributes 50 per cent of its earnings, after payment of $100,000 as salary, the tax burden of these men would be $174,000 less than under a partnership.[5]

There had been agitation for such a tax on undistributed profits for a long time. Secretary of the Treasury Houston, in 1920, advocated an undistributed profits tax, and Professor Adams of Yale argued for the undistributed profits tax in hearings before congress at that time. It is humorous to note that the National Association of Credit Men, who came out with a scathing denunciation of the law in 1938,[6] issued a pamphlet in 1920 urging that Congress should enact an undistributed profits tax as a substitute, much to be desired, for the excess profits tax.

Other proposals of a tax on undistributed profits, for the purposes of "Equalizing the taxation of incorporated and unincorporated business, and to equalize the taxation of large and small corporations, and to prevent the avoidance of the surtax," were made in 1924 (an amendment offered by Senator Jones of New Mexico to this effect passed the Senate, but was defeated in the House); in 1927 the National Tax Association advocated an undistributed profits tax, but no specific legislation was considered by Congress.[7]

These citations by no means exhaust the arguments of the proponents of the Undistributed Profits Tax. With many it was frankly a

[5] *Hearings on the Revenue Act of 1936* before the Committee on Ways and Means, 74th Congress, 2nd Session.

[6] *New York Times*, June 11, 1938.

[7] Buehler, *op. cit.*, pp. 7–14.

reform measure. American enterprise was so big it was unwieldy, and this, they argued, was in large part the cause of this most severe of all business crashes. We were overbuilt, overcapitalized—no one could expect to make a return on all that had been invested in the boom period. This tax would reduce the extent of corporate reinvestment on capital account, and restore a healthier balance. Further, when so much of the nation's purchasing power was going into capital goods, consumption could not keep pace. Corporate earnings plowed back into the business would not stimulate consumption nearly as much as would the payment of dividends. And even those who believed there should in general be no limit to corporate saving still regarded this tax as one way to stimulate consumption in depression, and therefore a desirable emergency measure.

It is, of course, quite beyond the scope of this paper to consider the general validity of the theory of business depression implicit in the above reasoning. But the extent of acceptance of this doctrine and the magnitude of corporate saving make very understandable the demand for undistributed profits tax legislation as an economic corrective measure. (In the four years 1926–1929, the approximately 400 corporations included in the Cowles Commission dividend and earnings indexes[8] "plowed back" more than 6 billion dollars out of a total of 14 billion earned on their common stock in the period, or roughly 45 per cent).

When, in 1938, Congress was considering the legislation which effectively modified the Undistributed Profits Tax out of existence, the Administration defended the tax on two grounds: It was claimed that the law was an important source of revenue, and must not be removed from the statutes without the substitution of other revenue legislation of commensurate yield. At the same time it was advanced that the measure aided the downtrodden small investor by increasing the distribution of earnings which, in the process of being plowed back into the business, are frequently plowed under.[9] Now it is possible to get

[8] Alfred Cowles 3rd and Associates, *Common Stock Indexes, 1871–1937*, Bloomington, Ind., Principia Press, 1938.

[9] An attempt to measure the loss to stockholders from the unprofitable reinvestment of corporate earnings is contained in the Cowles Commission study previously referred to (*op. cit.*, pp. 42–43). Here the conclusion is drawn that the typical investor receives, in the form of an increased price for the common stock he holds, only 70 per cent of the value of the undistributed earnings. While this percentage may overstate the stockholders' losses, because of a chronic corporate tendency to overestimate earnings in financial reports, there remains considerable ground for the claim that stockholder interests may be served by the increased distribution of earnings. Additional evidence on the question of the wisdom of reinvestment policy will be considered in a following paragraph.

at tax avoidance on the personal income side, and at the same time to compel the payment of dividends to small stockholders who are now frequently neglected, by making the Undistributed Profits Tax rates so severe that they will force distribution. But this means the corporations will distribute their earnings, and hence they will pay no surtaxes on them. On the other hand, if the rates are low enough not to make corporate saving prohibitive, they provide Undistributed Profits Tax revenue, but do not compel the distribution of earnings nor do they accomplish anything toward checking personal tax avoidance. Thus these two possible results of the legislation are mutually exclusive, at least on the assumption that no intermediate rate could achieve both increased corporate taxation and increased distribution of dividends.[10]

The question of the wisdom of corporate reinvestment appropriately requires consideration at this time. Some proponents of the Undistributed Profits Tax support it on exactly the same grounds which lead some opponents to reject it: namely, that by compelling distribution of nearly all earnings, it would force corporations to obtain all new funds in the organized capital market. An able study made from the point of view that compulsory resort to the capital market is desirable for expanding enterprises is presented by J. Ellwood Amos in his *Economics of Corporate Saving*.[11] With extensive documentation, Amos concludes that corporate "plowback" is more likely to be poorly invested than are borrowed funds because there is no investigation of such reinvestment by banks or investors; because the rate of interest, which weeds out low-return investment, is inoperative; and because

[10] In the first draft of this paper there appeared the statement: "The Administration supported the tax as a source of revenue, and as an aid in securing a wider distribution of earnings, yet it should be obvious that the attainment of one of these objectives substantially precludes the other." This statement is not justified, as several friendly critics were kind enough to point out upon reading the manuscript after its presentation at Detroit. My error lay, however, not in ignorance of the fact that some increase in personal-income-tax receipts is to be expected following corporate efforts at tax avoidance through increased earnings distribution, but rather in taking for granted the conclusion of this paper (that the tax did not materially increase dividend disbursements) in advance of reaching that conclusion. It must be remembered that the Administration's defense of the tax came after two years of experience with the specific legislation under review. The dual grounds on which the defense of the act was based are compatible only if it be assumed that the Undistributed Profits Tax *as enacted* was effective in compelling an important and substantial increase in the distribution of earnings. On the evidence of this study, that assumption seems untenable, and hence the Administration position inconsistent.

[11] *Illinois Studies in the Social Sciences*, Vol. 22, No. 2, 1937, Urbana, The University of Illinois, Chapter V.

corporation directors are almost unanimously of the conviction that "bigger means better." Among cited opinions on this point, that of David Friday is perhaps most succinct: "The volume of capital formation in no wise depends upon the wisdom or unwisdom of its investment."[12]

The opponents of the Undistributed Profits Tax were not, of course, innocent of contradictions. It was charged that the measure would raise corporation taxes exorbitantly. (This assumed that the tax would be paid, i.e., that the earnings would not be distributed.) It was also alleged that the law would discourage prudence, compel the distribution of every cent of earnings, leave nothing for a rainy depression, drive capital out of industry. (This argument assumed that earnings would be distributed; i.e., that the Undistributed Profits Tax would not be paid.)

Among recent popular criticism of the Undistributed Profits Tax that in the "Fortune Round Table" on Taxation[13] is rather surprisingly unsophisticated. The notion—discussed above[14] and considered in detail by Amos[15]—that corporate investment would be more soundly done if corporations had to secure their funds in the competitive capital market than it is when they simply withhold a portion of earnings from stockholders is dismissed as almost too ridiculous to merit comment. This position would have been much better defended had the "Round Table" authors taken the point of view held by Colm[16] that until the capital market is itself able to function more adequately, self-financing of expansion will be a necessity. There remains perhaps the unanswerable question to what extent corporations wishing to expand and unable to reinvest earnings might succeed in reviving the capital market. Here any opinion at all is surely hazardous.

Another astonishing statement in the "Round Table" discussion is the following remark: "Few of us, moreover, regard the theory of oversaving as having been established. Even if the theory were sound, it would not follow that a tax penalizing corporation *investment* would be justified."[17] Now the Undistributed Profits Tax penalizes the *retention* of corporate earnings, not their *investment*, and surely the phenomenon of so-called "reinvestment" taking the form of mere increase in the corporation's cash position is not so remote as to be unremembered.

[12] *American Economic Review*, Vol. 23, Supplement, p. 95.

[13] *Fortune*, May, 1939, pp. 113–114.

[14] Page 339.

[15] *Loc. cit.*

[16] Colm, Gerhard, "The Revenue Act of 1938" *Social Research*, Vol. 5, No. 3, September, 1938, pp. 276–277.

[17] *Fortune*, May, 1939, p. 114. Italics in original. Attention should be called to the harsh use of the subjunctive mood.

Further, the advocates of a tax on undistributed profits as a means of avoiding oversaving are not apologists for a naïve underconsumption theory of business cycles. Disciples of the new Keynes identify saving and investment, at least in the aggregate, but even among the great unwashed (i.e., the non-Keynesians) there is a large group which holds corporate saving excessive in that it leads to a diversion of too large a portion of the income stream into the purchase of capital goods, causing acute excess capacity.[18] Perhaps the right name for this is overinvestment rather than oversaving, but in that case the advocates of this tax must be credited (or charged) with an overinvestment theory of business depression.

If this maze of opinion is as contradictory as it appears under rather careful examination, the unanimity of acceptance of the conclusion that the Undistributed Profits Tax caused a great increase in the distribution of earnings is somewhat surprising. After the law had served its two years (1936–37) and been in effect repealed, those opposed to the tax wrote of the unwise distribution of earnings[19] while its advocates praised its benefit to the small stockholder. There appears to have been only one prominent dissenter to this inference. In 1936, before any data were available, Professor Lutz of Princeton predicted in the new edition of his text on public finance that the corporations of the country would not depart from established dividend policy, and, in so far as that policy increased their tax burdens, they would pay the taxes.

Few serious efforts to support by statistical evidence the widely held and expressed opinions seem to have been made. A notable exception (the Colm-Lehmann study) will be considered in the next section. The statistical analysis which follows is an attempt to give objective answer to the question: What were the results of this particular American experiment with an undistributed profits tax?

III

The data of this study are taken from the work sheets used in computing the Cowles Commission's *Common Stock Indexes, 1871–1937*. These indexes of prices, prices including the accumulation of cash dividends,[20] earnings, earnings-price ratios, dividends, and yields measure investment experience for 67 years. The price indexes are monthly, and include every issue quoted on the New York Stock Exchange for

[18] Note especially in this connection Haberler, *Prosperity and Depression*, Geneva, League of Nations, 1937, Chapters 3 and 5.

[19] See especially Sumner H. Slichter, "The Downturn of 1937," *Review of Economic Statistics*, August, 1938, pp. 100–101.

[20] Throughout this study, "dividend" refers to the cash payment made to holders of common stock, and "earnings" to the corporate profits available for distribution to these stockholders.

two or more consecutive months, 1871–1917, and more than 90 per cent in value of all common stocks on the Big Board for the next twenty years. The annual dividend indexes are equally inclusive, and the annual earnings indexes practically so, averaging 85 per cent coverage. The price, dividend, and earnings records of more than 1200 companies were analyzed. Over 400 companies appear in the indexes, in each of the recent years.

These records were obtained from *Moody's Manuals* and *Supplements*, and from the *Commercial and Financial Chronicle*, and include all of the companies in the price indexes for which earnings reports were available. An alternative source giving aggregate data on many more concerns, but not providing a breakdown into satisfactory industry subgroups is of course the *Statistics of Income* reported by the Federal Treasury. This source, which has been used by Amos,[21] was rejected for this study because net income for tax purposes may be very different from the net profit on the basis of which corporate directors plan dividend policy. It seems fair to suppose that what the directors report to their stockholders as net income is actually the figure upon which decisions are based. These reports are the ones summarized in the *Manuals* and *Chronicles*, and it is therefore upon them that our computations are made.

The computation of a dividend-earnings ratio may be based on the records of all reporting corporations, on the records of those companies showing positive earnings,[22] or on the records of only those concerns revealing earnings in excess of dividends.[23] The first method was the one employed in this study. It tends to measure (subject to the adequacy of the sample) the ratio of total dividends to total earnings for all industry. A corporation is privileged to pay out dividends as long as any surplus available for this purpose exists. Therefore the decision to pay dividends in excess of positive earnings and the decision to pay dividends despite losses are equally disinvestment, and they would seem to deserve equal treatment in a study of corporate distribution practice. On this argument, the exclusion of corporations simply because they fail to show positive earnings is unjustified. The discarding of dividend and earnings records of *all* corporations which disinvest (i.e., retaining only those companies with earnings in excess of dividends) is not subject to this criticism, but while this method of selection gives equal treatment to all cases in which part or all of dividends come from surplus, it excludes a great many corporations in a

[21] *Op. cit.*, pp. 31 ff.

[22] Colm, *op. cit.*, p. 277; Amos, *op. cit.*, Chapter III.

[23] Gerhard Colm and Fritz Lehmann, *Economic Consequences of Recent American Tax Policy*, New York, The Graduate Faculty of Political and Social Science, New School for Social Research, 1938, pp. 34–35.

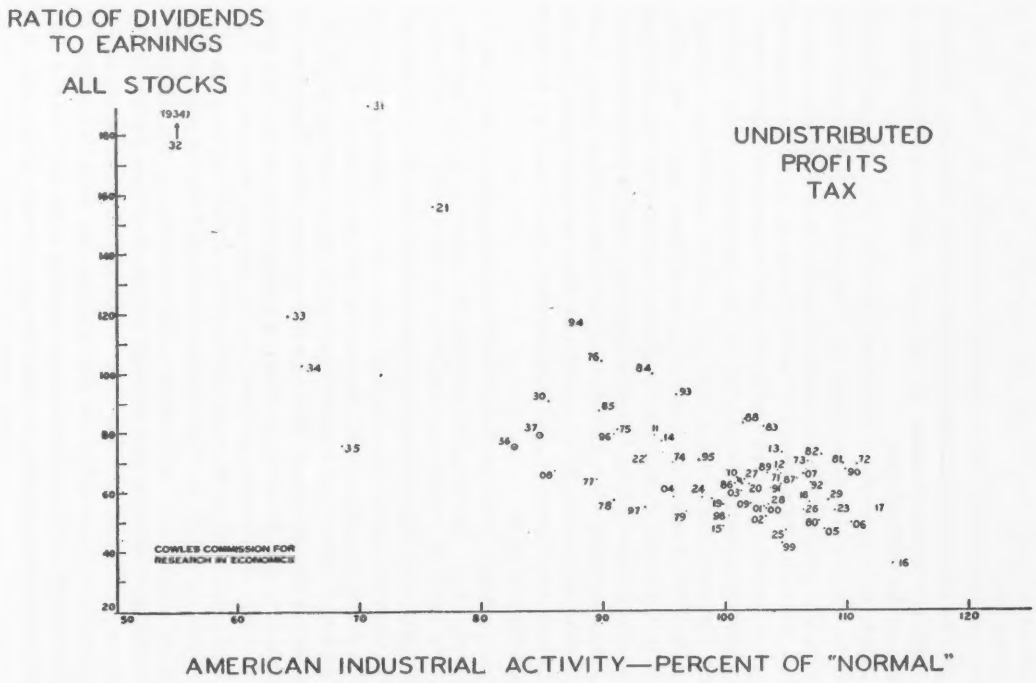
RATIO OF DIVIDENDS
TO EARNINGS
ALL STOCKS
UNDISTRIBUTED
PROFITS
TAX
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FIGURE 1.

position to save a great deal, and whose investment decisions are of major importance to all industry. Further, a haphazard element is introduced, because dividend decisions are usually made before earnings reports are complete, and a firm intending to distribute 90 per cent of (say) 1937 earnings may be excluded because an earnings report appearing in June, or even September, 1938 shows that the dividends were 105 per cent of earnings. Nevertheless, as a partial test of this approach, we calculated the percentage of earnings distributed to stockholders during each year of the decade 1928–1937 by those corporations in our sample which reinvested some portion of their earnings in the year in question. The results of this computation appear below, along with the results of the larger study including all reporting companies in the Cowles Commission indexes.

The question of an appropriate measure of the general level of business activity may also be raised. Since it was desired to employ such an index in ratio-to-trend form, the familiar Cleveland Trust index was used. This selection provides a long-period series which may be compared with the Cowles Commission indexes for the past 67 years.

When the ratio of total dividends to total earnings is plotted against the ratio of business activity to trend, as in Figure 1, the high negative association is at once apparent. In fact general industrial prosperity seems to be the most important single factor determining the dividend-earnings ratio. This tendency of corporations to reinvest most heavily during booms is neither new nor surprising. It is an important element in Colm's argument.[24] Yet it is apparently omitted from Amos' analysis of "Factors Affecting Corporate Savings."[25]

For all companies, the percentages of earnings distributed in 1936 and 1937 were near the average for the last 67 years. Twenty of the 67 years saw dividend-earnings ratios higher than 74 per cent—the 1936 figure. The 1937 percentage was just over 78 per cent. There were fourteen years, more prosperous than 1936, and without any undistributed profits tax, in which the ratio of dividends to earnings exceeded that of 1936. In brief, the law did not cause a substantial change in corporate dividend policy.

Since the Cowles Commission indexes are presented for Industrial, Rail, and Utility common stocks and 65 subclassifications under these headings, as well as for All Stocks combined, we may observe which industries show the greatest signs of having been affected by the Undistributed Profits Tax during the two years of its operation.

For the three broad types of companies we can say only that the continuing financial plight of the carriers makes conclusions impossible,

[24] *Op. cit.*, p. 278.
[25] *Op. cit.*, Chapter IV.

the Utilities were entirely unaffected, and the Industrials show some slight effect of the tax. Specific industries which evidently increased substantially their proportionate distributions on account of the tax are: Sugar Producing and Refining; Theatres; Silk and Silk Goods; Containers; and almost all subdivisions of Retail Trade. This last classification includes Drug Chains, Grocery and Meat Chains, Restaurant Chains, Mail Order Houses, and Department Stores.

A larger group of industries must be listed as "doubtful"—i.e., showing only slight effect if any. These include: Railroad Equipment; Miscellaneous Manufacturing; Electrical Equipment; Food Products; Office and Business Equipment; Tobacco Products and Cigarettes; Autos and Auto Parts; and Cotton.

Finally, we may say there is considerable evidence that the law had no aggregate effect upon the dividend policies of the following industries: All subdivisions of Utilities, whether operating, holding, traction, or telephone; Mining and Smelting; Steel and Iron; Household Products; Copper and Brass; Machinery and Machine Equipment; Chemicals; Oil Producing and Refining; Building Equipment; Drugs, Medicines, and Cosmetics; Radio; Beverages; Rayon.

These conclusions have been drawn after a study of the dividend-earnings ratios for each industry, with special reference to the configuration of points (like the one shown in Figure 1) and the position of the ratios for 1936 and 1937 in this configuration.

In order to compare the above conclusions for all companies with those which might be drawn from an analysis based entirely on corporations showing some reinvestment (i.e., dividends less than earnings), the following table was constructed:

TABLE 1[26]

Year	Number of Included Corporations	Total Earnings (000 omitted)	Total Dividends (000 omitted)	Ratio of Dividends to Earnings
1928	341	$3,606,476	$1,902,723	52.8%
1929	338	4,152,765	2,166,647	52.2%
1930	239	2,567,413	1,601,184	62.4%
1931	142	921,202	660,274	71.7%
1932	93	433,598	302,784	69.8%
1933	215	917,481	515,891	56.2%
1934	219	1,111,823	718,295	64.6%
1935	276	1,567,703	851,984	54.3%
1936	317	2,548,759	1,674,783	65.7%
1937	273	2,622,488	1,752,258	66.8%

[26] Based on all Industrial, Rail, and Utility corporations in Cowles Commission indexes which made earnings in excess of dividends for the year shown.

This is undoubtedly too short a period to permit the making of extensive generalizations, yet a few hesitantly advanced observations may be in order. There is a temptation to compare 1936 and 1937 with only the two immediately preceding years, and to conclude that an increasing ratio of disbursement, accompanying a rise in prosperity, proves the effect of the Undistributed Profits Tax. But the long period of low dividend payments (in dollars) undoubtedly influenced somewhat the decisions of directors to increase the dollar amount of distributions as much as possible, as soon as increased earnings had covered the bare minimum of reinvestment requirements.

Further, there were other real and imagined deterrents to the reinvestment of corporate funds, of which sufficient political capital has already been made without their mention here. Going back a few more years, it may surely be said that the ratios for 1936 and 1937 do not exceed those for 1928 and 1929 by more than the greater prosperity of the earlier years would lead one to expect. Also, the ratio for 1930 is higher than for either 1936 or 1937, although the Cleveland Trust index shows 1930 as a more prosperous year (measured in terms of the ratio of business activity to trend) than were either of the two years during which the Undistributed Profits Tax was a serious part of our fiscal structure.

IV

The conclusion that the Undistributed Profits Tax did not materially increase the distribution of corporate earnings is in conflict with popular opinion. If the elementary statistical analysis presented above, and the conclusions drawn from it, are valid, this conflict is probably due to the greater newsworthiness of an increase in dividends, as compared with a continuation of established policy. One item of publicity which was evidently misleading was the news that dividend payments in November of 1936 set a new high record for a single month.[27] This was interpreted by some writers[28] as proof of increased disbursements due to the tax; but it now appears more probable that this peak was due, not to an increase in the percentage of earnings which corporations intended to distribute, but rather to an advancement of the date of distribution. The Revenue Act of 1936 provided that the credit for dividends paid might be taken only for dividends distributed during the taxable year.[29] Thus the law compelled corporations which normally awaited the year's earnings report before declaring a final dividend (usually in February or March) to estimate the profitability of

[27] *New York Times*, Nov. 29, 1936.
[28] See, for example, Amos, *op. cit.*, pp. 117–118.
[29] Buehler, *op. cit.*, p. 213.

the year's operations as much as two months before the close of that year, in order to permit the declaration and payment of a dividend within the period of credit allowance.

It would appear, then, that economic historians and students of public finance ought to evaluate the recent American experience with a tax on the undistributed profits of corporations in terms of its efficiency or lack of efficiency as a revenue measure, and not as a reform measure. It evidently accomplished very little toward the reduction of corporate saving, in the aggregate, and the small investor was apparently no better off than before.

Cowles Commission for
Research in Economics

A NOTE ON IRVING FISHER'S CONCEPT OF INCOME*

By Clyde H. Graves

Professor Fisher in his paper "Income in Theory and Income Taxation in Practice"[1] claims that: (1) he has demonstrated[2] mathematically that such accruals as, e.g., capital gains, appreciation, and saving are "not properly income,"[3] and that (2) taxation of accruals of this order under an income tax modelled after the type of concept of taxable income which underlies the present federal and most state personal-income-tax statutes is productive of "double taxation."[4]

In the present note, an analysis will be made of the relation between Professor Fisher's assumptions and conclusions.

By way of defining some basic concepts apparently necessary to his demonstration Professor Fisher defines income as a "flow of services through a period of time" and capital as "quantity of wealth existing at a particular instant of time." Wealth, according to Fisher, consists of material objects owned by human beings. Services are the benefits of wealth. At a particular instant of time a person may hold title to a certain amount of wealth. In that event the person under consideration will receive services from his wealth during an interval of time. For example, a power plant is wealth, the electricity produced is service, and income is the amount of electricity, number of kilowatt-hours used over an interval of time. It is with the "correct" measurement of income in time that Professor Fisher is primarily concerned and it is for the purpose of income measurement that he constructs a meter whose mechanism bears investigation.

It is to be noted that Professor Fisher in "Income in Theory and Income Taxation in Practice" is not concerned with the changes in "capital" but only with "services" which are derived from capital. According to him, the two concepts are often confused, and "capital gain" is a change in "capital" and therefore not a "service," and therefore not income.

Using the notation introduced by Professor Fisher, let:

Y = Psychical services (satisfactions and dissatisfactions) produced by a certain quantity of wealth, measured in dollars, and received in a certain interval of time.

* This note was suggested by Dr. P. H. Wueller's paper "Concepts of Taxable Income II; The American Contribution," *Political Science Quarterly*, December, 1938.

[1] Fisher, I., "Income in Theory and Income Taxation in Practice," Econometrica, Vol. 5, No. 1, January 1937, p. 1 et seq.

[2] *Ibid.*, p. 47.

[3] *Ibid.*, p. 2.

[4] *Ibid.*, p. 2.

$\sum Y$ = Total psychical services received in the interval of time.
Y_p = Positive psychical services (satisfactions).
Y_n = Negative psychical services (dissatisfactions).
R = Money value of "real services" of consumption goods.
M_s = Money spent on consumption goods.
M_w = Money received from work (salary, wages, fees, commissions, etc.).
M_r = Money received in the interval of time from investments of property, including any proceeds of sales of property, gifts, bequests, loans—all money receipts whatever not included in M_w.
M_d = Money disbursed, including purchase price of property, interest, payment of loans—all money disbursements not included in M_s.
m = Money paid out from cash assets less money debited to cash.
U = Unclassified services.
I = Income received in the interval of time.

Professor Fisher's definition of income [the algebraic sum of all services received—his formula (21)] follows:

$$(1) \qquad I = (\sum M_r - \sum M_d) + (\sum M_w - \sum Y_n) + (\sum R - \sum M_s) + (\sum Y_p - \sum R) + m + U.$$

This is equivalent to

$$(2) \qquad I = \sum Y_p - \sum Y_n + U.$$

Assuming, as Professor Fisher does, that $U = 0$, income becomes *ipso facto* the algebraic sum of psychical services. Therefore [his formula (24)]

$$(3) \qquad I = \sum Y_p - \sum Y_n.$$

In order to measure I, Professor Fisher makes the following assumptions:

(1) Psychical services are measurable, at any rate *indirectly* (p. 31).

(2) The value (i.e., the measure) of the sum of negative psychical services is provisionally and for practical reasons to be zero (p. 33).

(3) The only psychical services which are to be included in the definition of income are those which correspond to objective services, i.e., there is to be one-to-one correspondence between Y items and R items (p. 33).

(4) The value of a Y item is "normally" equal to the corresponding value of the R item (p. 33).

(5) The money value of an objective service equals the money spent for the service.

Assumptions (4) and (5) imply that the money value of a psychical enjoyment equals the price paid for the objective service which presumably induces that enjoyment.

From these assumptions it follows that:

$$I = \sum Y_p - \sum Y_n = \sum Y_p = \sum R = \sum M_s.$$

In fine, the combined effect of the above assumptions is to equate income and consumption.

It may be asked what is to be gained by constructing the system of concepts in such a way that income is consumption? And, if such a system is wanted, we might construct it in a less elaborate way, simply by writing

$$I = \sum M_s.$$

If the above equation is accepted as an adequate definition of income it follows immediately that accruals of the order of capital gains are *by definition* excluded from the category of accruals which partake of the nature of income.

If we do not follow Professor Fisher's more elaborate method, the adequacy of his concept of income must be evaluated in terms of the conventions which he selects for the purpose of giving his definition content.

Why is it assumed that $\sum Y_n = 0$? There are no doubt many practical difficulties involved in measuring the value of "dissatisfactions" but probably no more than in measuring the value of "satisfactions."[5]

However, it is in connection with his postulation of (3) that Professor Fisher departs radically from contemporarily employed concepts of taxable income. It is coincident to the establishment of this convention which leads directly to the measurement of $\sum Y_p$ that Professor Fisher chooses to measure and count only those psychical services (satisfactions) which are coupled with and presumably causally related to so-called "objective services" which are evidenced by the destruction of consumption goods and which are measured by the market price of these goods. It has been pointed out previously that this deletion of what are commonly considered income constituents from the income category is inconsistent with Professor Fisher's basic doctrinal orientation[6] and contrary to the social objectives of personal income taxes.[7] For purposes of the present Note, however, it suffices to say that the

[5] Haig, R. M., "The Concept of Income—Economic and Legal Aspects," in *The Federal Income Tax*, New York, 1921, p. 1.

[6] See Wueller, P. H., "Concepts of Taxable Income II; The American Contribution," *Political Science Quarterly*, December, 1938.

[7] See Simons, Henry C., *Personal Income Taxation*, University of Chicago Press, 1938.

necessity for the exclusion has not been mathematically "demonstrated."

Without passing judgment upon Professor Fisher's concept of taxable income as outlined above as a tool of social policy, it is proposed to investigate his claim that incorporation of his definition into fiscal statute would eliminate present "double taxation."[8]

According to Professor Fisher "double taxation" is inevitable whenever a fiscal statute is patterned after a concept of income of the accrual type. Attempting to substantiate this claim Professor Fisher observes: "When a salaried man pays a tax this year on his whole salary, including what he saves and invests, and in future years pays taxes on the income then coming in from this year's savings, he is the victim of double taxation."[9] By double taxation Professor Fisher means that the present value of the tax on the income from the savings *equals* the income tax paid on that fraction of a given income which was originally devoted to savings purposes.[10] In other words Professor Fisher claims that if a given person receives in a given year an income of (say) $1000.00 and if out of that income he saves and invests $100.00, the present value of the future income-tax payments due on the interest from the $100.00 saved will *equal* the tax due on the $100.00 for the year during which it accrued as income, or, to use Professor Fisher's own terminology, for the year during which it accrued as "total gain."

Professor Fisher's contention is true only if it is assumed that (a) the capital saved will never be spent, and (b) that the income-tax rate, in Professor Fisher's terminology the rate of tax on "total gain," remains constant in time. However, it can be demonstrated that, if it be assumed that (a) either the tax rate on "total gain" will decline with the passage of time, or (b) that the capital sum saved will be dissipated at some future date, the present value of the successive tax payments due on the income from the savings will be *less* than the tax imposed upon the savings at the time when they accrued to a given individual as part of his "total gain."

Let:

J = "Total gain," that is, income as defined in accordance with concepts of income of the accrual type; in Fisher's terminology, "income before savings are taken out."

E = "Expenditure," that is, outpayments for consumption goods; in Fisher's terminology, "income after savings" or "income."

[8] See Fisher, I., *loc. cit.*, p. 48.

[9] *Ibid.*, p. 47.

[10] *Ibid.*, p. 47.

S = "Savings," that is, the difference between "total gain" and "expenditure."
r = Rate at which "total gain" is taxed annually, hereafter referred to as "income-tax rate."
j = Rate at which expenditures are taxed annually, hereafter referred to as "expenditure-tax rate."
i = Interest rate, compounded annually; that is, rate of return on savings.
n = Number of years over which capital sum saved is maintained intact.
t = Time, measured in years, taking the present as origin.
$v = (1+i)^{-1}$.
B = Present value of both "income" and "expenditure" taxes.

For purposes of investigating the effect of the taxes on S, it is assumed that in the future the income from S will all be spent and that it will be subject to both an income and an expenditure tax. It is further assumed that S will be spent in n years from date and when spent will be subject to an expenditure tax. These assumptions will make the value of B larger than if the expenditure tax on the income from S is calculated after deducting the income taxes paid. It is assumed that the interest rate is constant, but that both r and j change in time and that they are represented by linear functions of t.

Let

$$r = mt + k,$$

$$j = at + b,$$

where

m = slope of the trend line of the income-tax rate,
a = slope of the trend line of the expenditure-tax rate,
k = the income-tax rate at the present instant of time,
b = the expenditure-tax rate at the present instant of time.

The income from the savings each year will be Si. Since B is defined as the present value of the taxes paid, it follows that:

$$B = kJ + bE + \sum_{t=1}^{n} (mt + k)Siv^t + \sum_{t=1}^{n} (at + b)Siv^t + (an + b)Sv^n$$

$$= kJ + bE + Si\left(m\sum_{t=1}^{n} tv^t + k\sum_{t=1}^{n} v^t\right)$$

$$+ Si\left(a\sum_{t=1}^{n} tv^t + b\sum_{t=1}^{n} v^t\right) + (an + b)Sv^n$$

$$= kJ + bE + (an + b)Sv^n$$

$$+ Si\left[(m+a)\sum_{t=1}^{n} tv^t + (k+b)\sum_{t=1}^{n} v^t\right]$$

$$= kJ + bE + (an+b)Sv^n + Si(b+k)v\,\frac{1-v^n}{1-v}$$

$$+ Si(a+m)\left[(n+1)\,\frac{v^{n+1}}{v-1} - \frac{v}{v-1} - \frac{v^{n+2}}{(v-1)^2} + \frac{v^2}{(v-1)^2}\right]$$

$$= kJ + bE + S(an+b)v^n + S(b+k)(1-v^n)$$

$$+ S(a+m)\left[1 + \frac{1}{i} - (n+1)v^n - \frac{v^n}{i}\right].$$

The value of B will now be determined for a few special cases.

Case I. Assumptions:

1. The amount S is never spent.
2. The income-tax rate is constant ($m=0$).
3. The expenditure-tax rate is zero ($a=b=0$).

Then

$$B = kE + 2kS.$$

The above is the special case which leads to Professor Fisher's contention regarding the "double taxation" of savings. In the case under consideration it is true that the present value of the future income-tax payments due on the interest from the amount S saved will equal the tax due on S in the year during which it accrued as income.

One may, of course, object to such "double taxation" provided one is prepared to show that it is (1) either "unfair," or (2) represents an impediment to economic progress. The first charge is not arguable, unless and until the concept of equity is rigorously defined and agreed upon. The validity of the second charge is a function of one's views regarding possible patterns of progress. For the limited purposes of this paper it suffices to say that there would seem to be alternative patterns of progress[11] and that the role of savings as a contributory factor to progress varies as different patterns are subscribed to.

Case II. Assumptions:

1. The amount S is never spent.
2. The expenditure-tax rate is constant ($a=0$).
3. The income-tax rate is zero ($m=k=0$).

Then

$$B = bE + bS.$$

In this case the present value of the expenditure tax on the income from S is the same as the value of the tax on S if the amount S was used

[11] Colm, G. and Lehmann, F., *Economic Consequences of Recent American Tax Policy*, New York, 1938, pp. 3–9.

for purposes of expenditure in the year in which it accrued as income.

The above is the special case which leads Professor Fisher to conclude that an expenditure tax (called by him an "income tax") is to be preferred to a tax on "total gain" (commonly referred to as an income tax) because, according to Professor Fisher, a tax on expenditure does not discriminate against the "saving use" of a given time period's net accruals. It will be shown subsequently that this contention is justifiable only if it is assumed that $a=0$.

Case III. Assumptions:

1. The amount S is never spent.
2. The income-tax rate is constant ($m=0$).
3. The expenditure-tax rate is constant ($a=0$).

In this case

$$B = bE + bS + kE + 2kS,$$

and again, the "saving use" of a given time period's net accruals is taxed heavier than the "spending use," the degree of discrimination being a function of the income-tax rate.

Case IV. Assumptions:

1. The amount S is spent at the end of n years.
2. The income-tax rate is constant.
3. The expenditure-tax rate is constant.

Then

$$B = bJ + kE + kS(2 - v^n).$$

The effect of the income tax on savings is a function of the time period over which the savings are maintained intact, the differentiation against savings becoming accentuated as n increases but never exceeding the differentiation indicated under Cases I and III. The factor $(2-v^n)$ approaches 2 as n increases but for all values of n will be less than 2. The following illustration indicates how the length of the time period affects the value of the "discrimination factor."

For $i=.04$, $n=\ 10$, $(2-v^n)=1.3244$;
$n=\ 25$, $(2-v^n)=1.6249$;
$n=\ 50$, $(2-v^n)=1.8593$;
$n=100$, $(2-v^n)=1.9802$.

Though the above illustration indicates that the value of the discrimination factor is less than 2 even if savings are maintained intact for one hundred years, Professor Fisher might still maintain that under contemporary American income-tax practice savings are subjected to "double taxation."[12] Such insistence seems unfortunate since it may convey the impression that the value of the discrimination factor is *necessarily* 2.

[12] Fisher, I., *loc. cit.*, p. 47.

Case V. Assumptions:

1. The amount S is never spent.
2. The income-tax rate is zero.
3. The expenditure-tax rate is variable (linear function of t).

Then

$$B = bE + bS + aS\left(1 + \frac{1}{i}\right).$$

In this case, if the trend is downward, i.e., a is negative, the tax burden on S is less than in Case II and the effect of the tax instead of discriminating against saving will favor savings. However, if the trend is upward, saving would be discriminated against and one *might say* that under Fisher's definition of income there would be "double taxation" of savings. In other words, in the absence of certainty of rates in time even Professor Fisher's income base might discriminate against the "saving use" of a given time period's net accruals.

There are, of course, many assumptions that can be made concerning the behavior of the income-tax rate and the expenditure-tax rate. In general, one can say that with an increase in the rate of the tax, whether it be an income tax or an expenditure tax, the present value of the tax on the income from savings will be greater than the value of the tax if the rate of the tax is kept constant.

However, if the present value of the tax on the income from savings is to be an important factor in a policy recommendation regarding an income tax and an expenditure tax, a statistical study should be undertaken with a view of ascertaining in terms of some probability calculus (1) the length of the time interval during which savings are kept intact, (2) the effect on inheritance and gift taxes on savings, and (3) the trend of the tax rates. Given (1) the probable values of n and i and (2) agreement among policy makers regarding the value to be assigned to the present value of the tax on savings and the income from savings, rates r and j can then be determined so as to attain the aims of a given policy.

It is not the purpose of the present note to make any policy suggestions whatever. Its objective has been merely to show that the conclusions which emerge from Professor Fisher's analysis are implicitly determined by what he considers "desirable" policy. In addition an attempt has been made to show (see, e.g., Case V above) that even granting the general acceptability of Professor Fisher's view on what is "desirable," his ends are not necessarily attained by the means which he prescribes.

The Pennsylvania State College

THE CONCEPT OF INCOME: A REBUTTAL

By Irving Fisher

Before commenting on Dr. Graves' note, I wish here to put on record a recent change in my own terminology. Dr. Graves quotes me correctly as having formerly said that capital increase is "not properly income." But since my efforts thus to restrict the use of the term "income" to "services" have met with little success, I have decided to capitulate on that point. In a book which I am preparing on "What should be taxable income?" I am suggesting that, paradoxically, we may avoid ambiguity by giving two meanings to "income." One is "services" or "yield income" or simply "yield." The other is "services-plus-capital-income" or "enrichment income" or simply "enrichment" (sometimes called "accretion"—sometimes "earnings").

This concession is made in deference to the fact that usage vacillates between the two concepts, and that both are useful. They are well exemplified in the stock market in the "yield" of a stock as contrasted with its "earnings." I am also prompted to make this change by the hope of helping to lift the discussion of income from the level of words to the level of ideas.

Apart from terminology, however, I have seen no reason since 1897 to change my attitude. Nor do I find anything in Dr. Graves' discussion which would incline me to change it.

In summarizing my conclusions, Dr. Graves is likely to mislead many readers:

(a) They may not understand that in what he calls my "assumption (2)" negative psychic services were omitted explicitly in order to create a simplified case—just as he himself makes various assumptions to create his five special cases. In my Econometrica article, negative psychical services were far from overlooked or disregarded. But I was leading up to the problem of taxation in which negative psychical services have to be either neglected or treated by rule of thumb.[1]

(b) The same is true of his reference to my "assumption (5)" to the effect that services are worth their cost. In my article, much attention was given to the "discrepancies" between the worth of services or uses and money cost. But, with his eye on only my most simplified case, Dr. Graves remarks:

> It may be asked what is to be gained by constructing the system of concepts in such a way that income is consumption? And, *if such a system is wanted*,[2] we might construct it in a less elaborate way, simply by writing
>
> $$I = \sum M_i.$$

[1] See Econometrica, Vol. 5, January, 1937, p. 32.

[2] The italics are mine.

If the above equation is accepted as an adequate definition of income [it is not so accepted by me] it follows immediately that accruals of the order of capital gains are *by definition* excluded from the category of accruals which partake of the nature of income.

In the above passage and in other passages in his article, Dr. Graves seems to imply:

(1) that my "elaborate" analysis and "system" of concepts give no result except the above equation;

(2) that, therefore, I might obviously have started with that equation;

(3) that the whole procedure is a futility and a begging of the question;

(4) that it represents wishful thinking, setting up definitions for the purpose of proving a "wanted" result.

In any or all of these four interpretations of his criticisms I may be doing Dr. Graves an injustice. But, in the belief that many if not all of his readers will interpret him in precisely these ways, I shall endeavor to show that all four are incorrect.

Not simply the special case where $I = \sum M_s$ but the general case was definitely discussed. One end result, instead of $\sum M_s$, was my formula (29). This was presented as the best practical approximation—as follows:

$$I_{ba} = (\sum M_r - \sum M_d) + \sum M_w + m + c,$$

where I_{ba} stands for the "best approximation" to income (in the yield sense) during the year; M_r stands for any item of money received from property during the same year; M_d for any money disbursed for property; M_w for any money received for work; m for the depletion in cash balance during the year; and c for the total of all use-cost discrepancies—which total of discrepancies is later broken down into three chief parts, each with its special formula.

Not only does my analysis give several results other than the simplified one quoted by Dr. Graves, but, on the way to those results, it gives at every stage answers to the many intermediate questions as to what is yield income. Merely to say that yield income consists of "consumption" (meaning the gross yield of food, dwellings, and other goods yielding personal uses), would be to skip over most of the cases to which the concept of yield or flow of services is, and should be, applied—especially those innumerable money transactions antecedent not only to "consumption" ($\sum R$) but to "spending" ($\sum M_s$). In fact, the formula (29) above cited contains no reference either to "consumption" or to "expenditure" (except in the small corrective item c). The formula refers mainly to *receipts* of money.

If there is any special virtue in my analysis of income, it lies in its

applicability to these many sorts and combinations of services—services of production, of transportation, and of exchange—quite as truly as to consumption; also in its applicability, by double-entry bookkeeping, to a uniform and self-consistent articulation of these various applications.

Furthermore, if there is one misrepresentation of my views which I have deplored more than any other, it is this which represents my concept as being limited to "consumption." "Consumption" is, perhaps, the most significant result but it is not the only one; and it is most certainly a result and not the starting point.

These procedures, it seems to me, cannot possibly be characterized as futilities nor as question-begging nor as wishful thinking. They are precisely the same procedures which accountants worked out long ago, but which economists, with a few exceptions,[3] still neglect. These procedures of summation, applied to the one simple concept of service, necessarily result in the conclusions set forth in my article, not omitting the very essential conclusion that the mere summation of services never involves any element of capital increase.

In order to include capital increase in "income" we must deliberately *add it on*. We thereby arrive at "enrichment" income, as a new and compound concept. No one has shown, and, it seems safe to say, no one ever will or can show, how to frame the enrichment concept *first* and then reach yield income by subtracting capital increase. We can *calculate* in both directions—from yield to enrichment and from enrichment to yield; but we can not *derive concepts* in both directions. Yet there seems to be a not uncommon impression to the contrary; and this impression is reflected by some of the phrases in Dr. Graves' article. We must never lose sight of the fact that capital increase is a part of capital value, and therefore is derived by discounting services. Thus, even enrichment income, like yield income, is really built out of services, that is, out of net current services *plus* discounted future services.

Dr. Graves seems to be laboring under the mistaken impression that the enrichment concept is generally accepted by economists as the sole meaning of income, that it corresponds to common usage, and that it is capable of definition independently of the concept of service. I challenge him to show that any of these three propostions is true.

A service rendered by an asset belonging to a given person is an event desirable to that person and occurring by means of the asset. So-called "consumption" is only a special case of service. Even the term consumption is misleading, for it implies that destruction is an essential

[3] A notable exception is John B. Canning in his *The Economics of Accountancy*, the Ronald Press, 1929.

result of service. For instance, Dr. Graves speaks of the services rendered by so-called consumption goods—more familiarly known as their "uses"—as "evidenced by the *destruction*[4] of consumption goods." Destruction is a nonessential of use. It is a concomitant of some uses. It is not universal. The use of some goods, like food, necessarily destroys them. They are not only used but "used up"—literally consumed.[5] But the use of a house does not destroy it. The use of a riding horse may prolong its life. The only essential is Use. Thus, "consumption," in the sense of literal destruction, does not help our analysis at all. Only service helps.

As for wishful thinking, I plead not guilty. My work on income started when I was Professor of Mathematics and it has always been a cold-blooded mathematical analysis of services, followed wherever that analysis might lead. I was even surprised to find where it did lead. Whatever views I now have on social policy—and these are still largely tentative—were acquired many years later; and were acquired wholly without reference to my, or any other, analysis of income. Nor does Dr. Graves give any evidence whatever to substantiate the implication that my income concept was chosen in order to lead up to some preconceived or "wanted" conclusion. It is therefore astonishing to find him, at the close of his article, asserting that his "objective" is to show "that the conclusions which emerge from Professor Fisher's analysis are implicitly determined by what he [Professor Fisher] considers 'desirable' policy"!

I emphasize the above disclaimer because it appears that others besides Dr. Graves have made the same mistake. In fact, I suspect that he took his cue from Wueller whose writings, he says, "suggested" his own; and from Simons whom he also cites. Both Simons and Wueller had apparently jumped to the same wrong conclusions, Wueller[6] apparently having, in turn, taken his cue from Simons.[7] It is in this way that many myths are created almost out of whole cloth.

But the chief conclusion of Dr. Graves' article is that my views on double taxation are only partially correct. For he finds that, even when capital increase *is* taxed twice (under a tax on income in the enrichment sense), the tax burden is not always *exactly* double.

[4] The italics are mine.

[5] So are raw materials, though these are not "consumption" goods—i.e., not personal-use goods which Dr. Graves evidently meant.

[6] Referring to Wueller, Graves says that my "deletion of what are commonly considered income constituents from the income category is inconsistent with Professor Fisher's basic doctrinal orientation." What my basic doctrinal orientation is is not stated. I am one of those who would like to know.

[7] Referring to Simons, who regards income taxes as primarily a means of redistributing wealth, Graves says that my views are "contrary to the social objectives of personal income taxes."

There is here no dispute—only a misunderstanding. By "double taxation" I never meant that the income tax was necessarily twice as great, but simply that the income was twice taxed. If I am not greatly mistaken, "double taxation" usually means exactly that—"twice taxed," not "twice the tax." When any property, estate, income, or anything else is taxed by two states or by one state and the Federal Government or by the Federal Government and a foreign government, there is said to be double taxation even if the two taxes be far from equal.

Graves might have added that, even in his Case I, if the tax on the capital increase of a source is paid out of that source itself, there is left only a part of that increase to bear income for a future tax. In this case the tax will be less than doubled.

With Graves' mathematical analysis I have no fault to find. It is a model of method, especially as it begins with the most general case and proceeds to interesting special cases. There is nothing whatever in this analysis inconsistent with my own.

Yet, as I reflect on Dr. Graves' article, I have a feeling that there remains, in some intangible way, a wide gulf between us. This is not so much, if at all, a matter of emotional "orientation," nor of preference as to tax policy, nor of the mere use of words, nor of any differences in mathematics. It is apparently only a difference in emphasis. Yet I suspect it is an important difference.

Perhaps the best way to express this is to state what I myself would most like to emphasize. This is that capital value is discounted yield income but not discounted enrichment income.[8] To recognize this fact and its significance in economic theory and in tax theory is, surely, of supreme importance. And it gives a special significance to the double taxation of savings[9] even if such taxation is to be defended and adopted. It means, for instance, that the only ultimate source of any sort of income tax is yield income. Capital increase is not another source but the same source, transformed and disguised as to time. A tax on capital is merely a pre-tax on its yield.

Enrichment income is important for balancing the books in any one year. But yield income is, I am convinced, the more fundamental concept, in fact the most fundamental in economic science.

Yale University
New Haven, Connecticut

[8] Unless with qualifications requiring certain shifting in time of capital increase. See my "Are Savings Income?" *Publications of the American Economic Association*, Vol. 9, No. 1, April, 1908. These qualifications are important in view of the common impression expressed in the loose statement that the market value of a security is the capitalized value of its earnings ("enrichment").

[9] See my "Double Taxation of Savings," *American Economic Review*, Vol. 29, No. 1, 1939.

ON THE SIGNIFICANCE OF PROFESSOR DOUGLAS' PRODUCTION FUNCTION: A CORRECTION

By Horst Mendershausen

It was found that some statements made in the last two paragraphs of my article[1] are not warranted by the preceding analysis. These two paragraphs should be replaced by the following:

The constant k indicates whether output and labor changed in the same proportion ($k=1$) or not ($k\neq 1$). It shows whether the volume of output per unit of labor remained constant or varied during the period under observation. In our case, this ratio changed constantly; it increased. Thus the divergence of k from 1 may be taken as an indicator of an evolutionary change in the volume of output per unit of labor. The particular value of k is due to the fact that simultaneously the industry became more "capital intensive" ($\gamma > \beta$), *and* that the numerical values of the three logarithmic trend slopes were such that the differences [(20) and (21)] stood in the ratio of approximately 3:4. k may, therefore, be taken as a kind of measure of the *evolutionary* changes occurring in industry during the twenty-four years.

Colorado College
Colorado Springs

[1] "On the Significance of Professor Douglas' Production Function," Econometrica, Vol. 6, No. 2, April, 1938, p. 153.

I am indebted to Mr. Robert J. Landry of the Brookings Institution for drawing my attention to the shortcomings of the original formulation.

ANNOUNCEMENT OF THE PHILADELPHIA MEETING DECEMBER 27–30, 1939

The December meeting of the Econometric Society will be held in conjunction with those of the other social science associations at Philadelphia, Pennsylvania, Wednesday through Saturday, December 27–30, 1939. Joint sessions with the American Statistical and Economic Associations and the Institute of Mathematical Statistics are planned for these meetings. A detailed program will be sent to each member of the Society in the United States and Canada early in December. Any inquiries regarding these sessions should be addressed to Francis McIntyre, Secretary of the Program Committee, Econometric Society, Stanford University, California.

ERRATA, JULY ISSUE

In "The Practice of Depreciation," by Gabriel A. D. Preinreich, ECONOMETRICA, Vol. 7, July, 1939, p. 253, notes 33, 34, and 35, the references "*op. cit.*" and "*ibid.*" are incorrect because of an error made by the managing editor. Footnotes 33 and 35 should refer to Kurtz, *Life Expectancy of Physical Property*, while footnote 34 should refer to Preinreich, "Annual Survey of Economic Theory: The Theory of Depreciation," ECONOMETRICA, Vol. 6, July, 1938, pp. 219–241.

Because of delay in the mail, proof of "The Demand for Passenger Cars in the United States: A Rejoinder," by P. de Wolff, ECONOMETRICA, Vol. 7, July, 1939, pp. 277–282, was not received from the author before the issue went to press. The following corrections should be made:

p. 279, para. 3, *for* Mr. Solo is convinced that . . . *read* It is not necessary to discuss in detail Mr. Solo's objections against formula (1), because he is convinced, on economic grounds, that . . .

p. 280, line 1, *for* whether $\epsilon > 1$ or $\epsilon < 1$ *read* whether $\epsilon < 1$ or $\epsilon > 1$.

p. 280, line 13, *add footnote*: When this article had already gone to press I received a report of the joint meeting of the American Statistical Association and the Econometric Society in Detroit, on December 27, 1939, containing a number of studies on the dynamics of automobile demand. One of these studies, by Messrs. Roos and von Szeliski, treats the same problem as has been dealt with in my own paper. Although their results in many respects are different from mine, it is interesting to note that they too find an elasticity of demand, with respect to price, greater than 1. The same report contains a study of Mr. DuBrul on the cost structure of the motor-car industry, in which it is

shown why a decrease in car prices, assuming constant factor prices, in general will lower profits, even in the case of elastic demand.

p. 280, para. 2, line 3 from bottom, *for* P_{01} *read* p_{01}.

p. 281, para. 2, line 4, *for* $p_{2k}' = p_{2k}/p_{00}$, we have, *read* $p_{ik}' = p_{ik}/p_{00}$, we have, by taking logarithms and again expanding the right-hand member in a power series:

p. 281, para. 2, line 5, *for P read* Log *P*.

p. 282, last line, for *The Hague, Netherlands,* read *Stockholm.*

Because of delay in the mail, proof of the note on Oskar Lange's election as a Fellow, in ECONOMETRICA, Vol. 7, No. 3, July, 1939, p. 284, was not received from Professor Lange before publication. It should be added to that note that Professor Lange also taught at the University of Cracow.

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AITCHISON, MISS BEATRICE, Dept. of Economics, University of Oregon, Eugene, Oregon.

AKERMAN, DR. JOHAN, University of Lund, Lund, Sweden.

ALIZON, DR. JULIO, Oficina General de Estadistica, Calle San Martin 1046, Rosario, Argentina.

ALLEN, MR. R. G. D., London School of Economics, Houghton Street, Aldwych, London, W. C. 2., England.

ALSBERG, DR. CARL L., Giannini Foundation, University of California, Berkeley, California.

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ANDERSON, PROF. M. D., University of Florida, Gainesville, Florida.

ANDERSON, PROF. OSKAR N., Director of the Statistical Institute for Economic Research, State University, Shipka 6, Sofia IV, Bulgaria.

ANDREW, MR. SEYMOUR L., American Tel. & Tel. Co., 195 Broadway, New York City.

ANGELL, PROF. JAMES W., Columbia University, New York City.

ANTONELLI, PROF. ÉTIENNE, Faculté de Droit de Montpellier, Montpellier, France.

AOYAMA, MR. HIDEO, Faculty of Economics, Kyoto Imperial University, Kyoto, Japan.

ARON, DR. ARNO, Grenchen, Kanton Solothurn, Switzerland.

AUPETIT, DR. ALBERT, 2 avenue Montaigne, Paris VIII, France.

AXE, MR. E. W., 730 Fifth Avenue, New York City.

AYRES, COL. LEONARD P., Cleveland Trust Co., Cleveland, Ohio.

AZMI, MR. HAMED EL-SAYED, 10 Khandak Street, Pont-de-Koubbeh, Egypt.

BACHI, PROF. RICCARDO, Via Antonio Bosio N. 2, Roma, Italy.

BAGGE, PROF. GOSTA, Stockholms Högskola, Stockholm, Sweden.

BAINOFF, DR. IVAN, 28 rue Graf Ignatieff, Sofia, Bulgaria.

BAKKER, DR. O., Dedelstraat 18, 's Gravenhage, The Netherlands.

BALKRISHNA, PRINCIPAL, Rajaram College, Kolhapur, India.

BANOS, PROF. OLEGARIO, Villa Lourdes, Haro (Logrono), Via Hendaye, Spain.

BARBIERI, MR. GINO, Università Cattolica del S. Cuore, Piazza S. Ambrogio 9, Milano, Italy.

BARDET, M. GÉRARD, 145 avenue du Général Michel Bizot, Paris XII, France.
BARFOD, MR. BØRGE, Tietgens Pl. 4, Aarhus, Denmark.
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BATHGATE, MR. ARTHUR E., Teulon, Manitoba, Canada.
BATTUELLO, DR. MICHELE, Officer of the Unfiel, Via Quattro Fontane 143, Roma, Italy.
BAYKITCH, M. VELIMIR, 59 rue Sanje Zivanovica, Beograd, Yugoslavia.
BEEN, MR. R. O., U.S. Bureau of Agricultural Economics, Washington, D.C.
BELCHER, MR. DONALD R., Asst. Chief Statistician, American Tel. and Tel. Co., 195 Broadway, New York City.
BELSCHNER, MISS MARTHA C., 1342 N. El Paso St., Colorado Springs, Colorado.
BELZ, MR. M. H., The University of Melbourne, Carlton N. 3, Victoria, Australia.
BENTLEY, DR. A. F., Paoli, Indiana.
BERNARDELLI, MR. H., 20 Pagan Road, University Estate, Rangoon, Burma.
BESSO, ING. M.A., 23 Chemin du Point du Jour, Petit Saconnex, Genève, Switzerland.
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BICKERDIKE, MR. C. F., 12 Hengrave Road, Forest Hill, London, S. E. 23, England.
BILIMOVIC, PROF. AL., Univerza Aleksandra, Ljubljana, Yugoslavia.
BISSELL, MR. RICHARD M., 1292 Yale Station, New Haven, Connecticut.
BJORAA, MR. S. J., Actuary of the Life Insurance Company "Andvake," 30 Stortings gt., Oslo, Norway.
BLAIR, MR. WILLIAM, 11 Ettnich Road, Edinburgh, Scotland.
BLAKSTAD, MR. EGIL, Chief Engineer, De No Fa, Fredrikstad, Norway.
*BODE, DR. KARL, St. John's College, Cambridge, England.
BOEHM, DR. CARL, Gustloffplatz 2, Berlin-Charlottenburg 5, Germany.
BOGART, PROF. ERNEST L., 35 East 9th Street, New York City.
BOLZA, DR. H., Postamt 7, Würzburg, Germany.
BONINSEGNI, PROF. P., Université de Lausanne, Lausanne, Switzerland.
BOOKER, MR. H. S., Asst. Lecturer in Statistics, London School of Economics, Houghton St., Aldwych, London W.C. 2, England.
BORDIN, PROF. ARRIGO, V. Fabro 2, Università, Torino, Italy.
BOREL, PROF. ÉMILE, Faculté des Sciences, Université de Paris, Paris, France.
BORGATTA, PROF. GINO, N. 1, Piazzale Baracco, Milano, Italy.
BOULDING, MR. KENNETH, Dept. of Economics, Colgate University, Hamilton, New York.
BOUNIATIAN, DR. MENTOR, 84 Avenue Victor Hugo, Paris XVI, France.
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BREIT, MR. MAREK, Koszykowa 45m. 19, Warszawa I, Poland.
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BRULÉ, M. PAUL E., Avenue Armand Huysmans 28, Bruxelles, Belgium.
BRYSON, MR. A. E., Vice-President, Halsey, Stuart Co., Inc., 201 S. La Salle Street, Chicago, Illinois.
BUCKLAND, MR. WILLIAM R., 131 Somervell Road, Harrow, Middlesex, England.
*BUFANO, DR. GIUSEPPE, Viale Gorizia, 43, Roma, Italy.
BURGESS, DR. ROBERT W., 440 Pelham Manor Road, Pelham Manor, New York.
BURKHARDT, DR. FELIX, Universität, Paulinum B., Leipzig, Germany.
BUSHEY, DR. J. HOBART, Hunter College of the City of New York, 68th Street and Lexington Avenue, New York City.
BYÉ, PROF. MAURICE, Faculté de Droit, 15 rue de Languedoc, Toulouse, France.

CAIRNCROSS, MR. A. K., Department of Political Economy, University, Glasgow, Scotland.
CAMPBELL, MR. C. D., School of Social Sciences and Administration, Department of Commerce, 19 Abercromby Square, Liverpool, England.
CANNING, PROF. JOHN B., Box 76, Stanford University, California.
*CARMILLE, M. RENÉ, 30 avenue Brimborion, Sèvres (Seine et Oise) France.
CARVER, PROF. HARRY C., University of Michigan, Ann Arbor, Michigan.
CHAIT, MR. B., 38 rue Gysels, Anvers, Belgium.
CHAMBERLIN, PROF. EDWARD, Dept. of Economics, Harvard University, Cambridge, Massachusetts.
*CHAMPERNOWNE, MR. D. C., King's College, Cambridge, England.
CHANG, MR. C. C., Bank of China, Shanghai, China.
*CHAO, DR. J. T., Professor of Economics, Department of Economics, Tsing Hua University, Peiping, China.
CHAYROU, INTENDANT GÉNÉRAL, 6 rue Rosa Bonheur, Paris XV, France.
CHEN, DR. WARREN, Directorate of Statistics, National Government, Nanking, China.
CHERINSKY, MR. ELMER, 505 E. 93rd Street, Brooklyn, New York.
CHUDNOWSKY, MR. W. A., Dept. of Economics, Yale University, New Haven, Connecticut.
CIRIACY-WANTRUP, PROF. SIEGFRIED VON, College of Agriculture, University of California, Berkeley, California.
CLARK, MR. COLIN, The Bureau of Industry, 108 George Street, Brisbane, Queensland, Australia.
CLARK, PROF. J. M., Columbia University, New York City.
COATS, MR. R. H., Dominion Statistician, Dominion Bureau of Statistics, Ottawa, Ontario, Canada.
COBB, PROF. CHARLES W., Amherst College, Amherst, Massachusetts.
COENEN, DR. P. A., C. Reinierszkade 25, 's Gravenhage, The Netherlands.
COHEN, MISS RUTH, Agricultural Economics Research Institute, Oxford, England.
COLEMAN, DR. GEORGE W., Securities Analysis Dept., Mississippi Valley Trust Co., St. Louis, Missouri.
COLM, PROF. DR. G., 100 Buckingham Rd., Yonkers-Nepperhan, N. Y.
COMPTON, DR. WILSON, 2900 Cathedral Avenue, Washington, D. C.
COOL, MR. WOUTER, 23 Prinsessegracht, 's Gravenhage, The Netherlands.
COURT, MR. ANDREW TRAWICK, Automobile Manufacturers Association, General Motors Bldg., Detroit, Michigan.
COURTIN, PROF. RENÉ, Professeur d'Économie Politique, Faculté de Droit de Montpellier, Montpellier, France.
COVER, PROF. JOHN H., Bureau of Foreign and Domestic Commerce, 50-f Dept. of Commerce, Washington, D.C.

COWLES, MR. ALFRED, Cowles Commission for Research in Economics, The University of Chicago, Chicago, Illinois.

CRACCO, PROF. FRANÇOIS, 32 boulevard de Tirlemont, Louvain, Belgium.

CRAIG, MR. JAMES IRELAND, Financial Secretary, Ministry of Finance, Cairo, Egypt.

CRAMER, PROF. HARALD, Skarviksv. 7, Djurholm, Sweden.

CRATHORNE, DR. A. R., University of Illinois, Urbana, Illinois.

CREEDY, PROF. F., 307 Hillsboro Street, Chapel Hill, N. Carolina.

CROXTON, PROF. FREDERICK E., Dept. of Statistics, Columbia University, New York City.

CRUM, PROF. WILLIAM LEONARD, 325 Littauer Center, Cambridge, Massachusetts.

DALMULDER, DR. J. J. J., Westzeedijk 76, Rotterdam, The Netherlands.

DANSON, MR. FORREST M., Cowles Commission for Research in Economics, The University of Chicago, Chicago, Illinois.

DARMOIS, PROF. G., 7 rue de l'Odéon, Paris VI, France.

DARRIEUS, ING. M. G., 7 avenue du Général Maunoury, à Houilles (Seine-et-Oise), France.

DASCOVICI, PROF. N., l'Université de Jassy, Iasi, Rumania.

DAVENPORT, DR. DONALD H., Securities and Exchange Commission, Annex 2, 1424 K Street, N.W., Washington, D.C.

DAVID, MISS FLORENCE N., Department of Statistics, University College, London, W.C.1., England.

DAVIES, PROF. G. R., College of Commerce, University of Iowa, Iowa City, Iowa.

DAVIES, MR. THURSTON J., President, Colorado College, Colorado Springs, Colo.

DAVIS, PROF. H. T., Dept. of Mathematics, Northwestern University, Evanston, Illinois.

DE ARTIGAS, PROF. JOSÉ ANTONIO, c/o The National City Bank of New York, 60 avenue des Champs Élysées, Paris VIII, France.

DE BOÉR, DR. ALEXIS, Kivalyhago u 5/A, Budapest I, Hungary.

DE FELLNER, PROF. FREDERIC, Palatin Joseph University, Budapest, Hungary.

DE KANTER, MR. ABRAHAM, Flatgebouw Willemspark, Zeestr. 73, 's Gravenhage, The Netherlands.

DE LANGEN, MR. W. J., Duinvoetlaan 5, Wassenaar, The Netherlands.

DE LOOR, DR. BAREND, Lynwoodweg 109, Pretoria, South Africa.

DEL VECCHIO, PROF. GUSTAVO, R. Università, Via delle Lame 8-10, Bologna, Italy.

DEMARIA, PROF. GIOVANNI, Università Commerciale Luigi Bocconi, Milano, Italy.

DERKSEN, DR. J. B. D., van Hoytemastraat 23 A, 's Gravenhage, The Netherlands.

DE PONTEVES, PROF. CHARLES, 31 parc de Montretout, Saint Cloud, (Seine et Oise), France.

DESSAU, MR. EINAR, Director Tuborg Brewery, Hellerup, Denmark.

DE SZADECZKY-KARDOSS, PROF. TIBOR, University, Debrecen, Hungary.

DE WOLFF, MR. P., Central Bureau of Statistics, 's Gravenhage, The Netherlands.

DE WOLFF, DR. S., Rubensstraat, 93, Amsterdam (Z), The Netherlands.

d'HARCOURT, M. JACQUES, 71 faubourg Saint-Honoré, Paris VIII, France.

DICKENS, MR. ALBERT E., Dept. of Economics and Statistics, Federal Housing Administration, Washington, D.C.

DIEULEFAIT, PROF. CARLOS E., 1235 rue Pueyrredon, Rosario, Argentina.

DIVISIA, PROF. FRANÇOIS, 95 boulevard Jourdan, Paris, XIV, France.

DODD, PROF. EDWARD L., 3012 West Avenue, Austin, Texas.

DODGE, MR. HAROLD F., Bell Telephone Laboratories, 463 West St., New York City.

DOMINEDO, PROF. VALENTINO, Via Carlo Alberto, Roma, Italy.

DONHAM, DEAN WALLACE B., Graduate School of Business Administration, Harvard University, Soldiers Field, Boston, Mass.
DONOVAN, COLONEL WILLIAM J., 2 Wall Street, New York City.
DORFMAN, MR. ROBERT, 145 West 79th Street, New York City.
DOUGLAS, PROF. PAUL H., University of Chicago, Chicago, Illinois.
DOYEN, M. PIERRE, Ingénieur de Ponts et Chaussées, Le Mans, Sarthe, France.
DUBRUL, MR. STEPHEN M., General Motors Corporation, Detroit, Michigan.
DUCASSÉ, M. F. A., 127 rue du Marche, Bruxelles, Belgium.
DUMONT, ING. AUGUSTIN, 80 rue de Monceau, Paris VIII, France.
DUMONT, MR. JEAN BENJAMIN, 202 Singel, Dordrecht, The Netherlands.
DUNCAN, PROF. A. J., 137 Jefferson Road, Princeton, New Jersey.
DUNNIGAN, MR. WILLIAM, 2952½ Leeward Avenue, Los Angeles, California.
DUPRIEZ, PROF. LÉON H., 145 rue des Moutons, Louvain, Belgium.
DUTILH, MR. J. R., 46 boulevard St. Michel, Paris, France.

EDELBERG, MR. VICTOR, 22 Temple Fortune Hill, London, N.W.11, England.
EDWARDS, PROF. RAMÓN SALAS, Dieciocho 277, Santiago, Chile.
EINAUDI, PROF. LUIGI, via Lamarmora, 60, Torino, Italy.
EISENHART, MR. C., Department of Mathematics, 206 North Hall, Madison, Wisconsin.
ELSAS, DR. MORITZ JOHN, 29 Steeles Road, London, N.W. 3, England.
ENGLIS, PROF. KAREL, Governor, National Bank of Czecho-slovakia, Praha, Czecho-slovakia.
EVANS, PROF. G. C., University of California, Berkeley, California.
EZEKIEL, DR. MORDECAI, Secretary's Office, Department of Agriculture, Washington, D. C.

FABRICANT, MR. SOLOMON, National Bureau of Economic Research, 1819 Broadway, New York City.
FANNO, PROF. MARCO, Via Aleardi 19, Padova, Italy.
FEILEN, DR. JOSEF, Institut fur Konjunkturforschung, Fasanenstrasse 6, Berlin-Charlottenburg, Germany.
FELLNER, DR. WILLIAM, Benczur utca II, Budapest, VI, Hungary.
FENCHEL, DR. WERNER, Sønderengen 55, Søborg, København, Denmark.
FERGUSON, MR. EDWARD W., 4338 Bruner Avenue, New York City.
FERRARA, DR. COSTANZA, Istituto Nazionale delle Assicurazioni, Roma, Italy.
FERRARI, PROF. GIOVANNI, Scuola di Statistica, Regia Università di Padova, Padova, Italy.
FERRARI, DOTT. RAG. GUIDO, Via Fratelli Bronzetti 28, Milano, Italy.
FICHTNER, DEAN CHARLES C., 209 Commerce Building, University of Arkansas, Fayetteville, Arkansas.
FISHER, MR. ARNE, Room 2101, 60 Hudson Street, New York City.
FISHER, MR. ERNEST M., Room 400, Federal Housing Administration, Washington, D.C.
FISHER, PROF. IRVING, 460 Prospect Street, New Haven, Connecticut.
FLANDERS, MR. RALPH E., Box 477, Springfield, Vermont.
FLOOD, DR. MERRILL M., Princeton University, Princeton, New Jersey.
FLUX, SIR ALFRED, c/o Royal Statistical Society, 4 Portugal Street, London, W.C. 2, England.
FONTIGNY, MR. PAUL, Zolder, Limburg, Belgium.
FORTUNATI, PROF. PAOLO, dell' Instituto di Statistica, R. Università de Palermo, Palermo, Italy.
FRANCK, DR. L. ROSENSTOCK, 55 rue de Lille, Paris, France.

FRANKEL, MR. LESTER R., Division of Research, W.P.A., 1734 New York Avenue, N.W., Washington, D.C.

FRANKEL, PROF. S. HERBERT, University of the Witwatersrand, Johannesburg, S. Africa.

FRAZER, MR. GEORGE E., Board of Trade Building, 39th Floor, Chicago, Illinois.

FRÉCHET, PROF. MAURICE, Faculté des Sciences de l'Université de Paris, 11 rue Pierre-Curie, Paris V°, France.

FREEMAN, MR. H. A., Massachusetts Institute of Technology, Cambridge, Massachusetts.

FRERICHS, ING. CHARLES, 21 rue Gachard, Bruxelles, Belgium.

FRICKEY, DR. EDWIN, 42 Holyoke House, Harvard Square, Cambridge, Massachusetts.

FRIDAY, DR. DAVID, 4525 Garfield Street, Washington, D. C.

FRIEDRICH, DR. W. G., c/o Mr. T. H. Marburg, Pine Top, Newfoundland, New Jersey.

FRISCH, PROF. RAGNAR, Vinderen, Oslo, Norway.

FRUMENTO, DR. ARMANDO, dell' Università Commerciale Luigi Bocconi, Viale Lazio 16, Milano, Italy.

FRY, DR. THORNTON C., Bell Telephone Laboratories, 463 West Street, New York City.

FULLER, MR. KEMP G., U. S. Steel Corporation of Delaware, 436 Seventh Avenue, Pittsburgh, Pennsylvania.

FUNKHOUSER, MR. RICHARD L., 3 Valley Road, Hanover, New Hampshire.

FURLAN, DR. LOUIS-VLADIMIR, Professor at the University, Basle, Switzerland.

FUSI, NOB. DEI BARONI DOTT. FRANCO, Via Petrarca 15, Milano, Italy.

GADOLIN, DR. CARL A., Nylandsgaten 1, Abo, Finland.

GAITSKELL, MR. HUGH, University College, Gower Street, London, W.C.I. England.

GALLIOT, M. A., 10 rue César Franck, Paris XV, France.

GARINO CANINA, PROF. ATTILIO, Corso Francia 100, Torino, Italy.

GARVER, PROF. FREDERICK B., 116 Vincent Hall, University of Minnesota, Minneapolis, Minnesota.

GEARING, MR. HAROLD W. G., 16 Drayton Gardens, London W. 13, England.

GEORGESCU-ROEGEN, DR. NICHOLAS, Strada Bitolia 5, Bucuresti III, Rumania.

GHEORGHIU, MR. DIM. I., Boulevard Vintilla, Brătianu 47, Bucuresti, Rumania.

GIBLIN, PROF. L. F., University, Melbourne N. 3, Australia.

GIBRAT, DR. ROBERT, 105 rue du Ranelagh, Paris XVI, France.

GIFFORD, MR. C. H. P., 32 Stafford Street, Edinburgh 3, Scotland.

*GILBOY, DR. ELIZABETH W., 40 Holyoke House, Cambridge, Mass.

GINI, PROF. CORRADO, Istituto di Statistica e Politica Economica, R. Università, Via delle Terme di Diocleziano, 10, Roma, Italy.

GLOERFELT-TARP, MR. B., Supervisor, Great Northern Telegraph Co., København, Denmark.

GOBBI, PROF. ULISSE, Università Commerciale Luigi Bocconi, Milano, Italy.

GOLDSCHMIDT, DR. R. W., c/o Securities and Exchange Commission, Analytical and Technical Division, 1778 Pennsylvania Ave., Washington, D. C.

*GOLICK, MR. A. FRANK, 30 North Curtis St., Chicago, Illinois.

GOLOVIN, MR. NICHOLAS, c/o R. H. Macy & Co., 34th and Broadway, New York City.

GONZALEZ, DR. RICHARD J., Room 1365, Humble Building, Houston, Texas.

GOUDRIAAN, PROF. J., JR., High School of Commerce of Rotterdam, Einhoven, The Netherlands.

GRAFÉ, ING. PAUL, 32 avenue de l'Indépendance Belge, Koekelberg-Bruxelles, Belgium.

GRAFF, PROF. F. W., John Carroll University, Cleveland, Ohio.

GRANT, PROF. EUGENE L., Department of Civil Engineering, Stanford University, California.

GRAZIANI, PROF. AUGUSTO, via Andrea D'Isernia, 4, Napoli, Italy.

GREBE, DR. JOHN J., 612 East Pine, Midland, Michigan.

GRIFFIN, PROF. F. L., Department of Mathematics, Reed College, Portland, Oregon.

GRIFFIN, DR. JOHN I., Dept. of Economics, Brooklyn College, Brooklyn, New York.

GROVE, DR. CHARLES C., 143 Milburn Avenue, Baldwin, Long Island, New York.

GUARESTI, DR. JUAN JOSÉ, Paseo Colón 185, Buenos Aires, Argentina.

GUMBEL, PROF. E. J., 1 rue P. Huvelin, Lyon, France.

GUTHMANN, PROF. HARRY G., Northwestern University, Commerce Building, Evanston, Ill.

HAAVELMO, MR. TRYGVE, The University Institute of Economics, Oslo, Norway.

HABERLER, DR. GOTTFRIED, Department of Economics, Harvard University, Cambridge, Mass.

HADAMARD, PROF. JACQUES, l'École Polytechnique, 25 rue Jean Dolent, Paris VII°, France.

HAGSTROEM, DR. K.-G., Actuary, "Framtiden," Stockholm 3, Sweden.

HALL, MR. N. F., The National Institute of Economics and Social Research, 32 Gordon Square, London, W.C.1, England.

HALL, MR. R. L., Trinity College, Oxford, England.

HAMBURGER, DR. IR. L., c/o Mr. Julius M. Eisendrath, 912-914 Empire State Building, New York City.

HAMILTON, PROF. THOMAS R., Box 204, Faculty Exchange, College Station, Texas.

HAMMARSKJÖLD, DR. DAG., Finansdepartementet, Stockholm, Sweden.

HANAU, DR. ARTHUR, Ithweg 8, Berlin-Zehlendorf, Germany.

HANDLEY, MR. LAWRENCE M., Investment Dept., Henry Phipps Estate, 465 East 57th Street, New York City.

HANSEN, MR. A. S., Consulting Actuary, Board of Trade Building, Chicago, Ill.

HANSEN, PROF. ALVIN H., Graduate School of Public Administration, Harvard University, Cambridge, Mass.

HANSON, MISS ALICE, 4406 38th Street N., Arlington, Virginia.

HARROD, MR. R. F., Christ Church, Oxford, England.

HARTKEMEIER, PROF. HARRY PELLE, 116 Business and Public Administration Building, University of Missouri, Columbia, Missouri.

HAWTREY, MR. R. G., 18 Elm Park Gardens, London, S. W., England.

HAYAKAWA, PROF. M., Midori-cho, 2 chome, Otaru-shi, Hokkaido, Japan.

HAYEK, PROF. DR. FRIEDRICH A., London School of Economics, Houghton Street, Aldwych, London, W.C.2, England.

HAYES, DR. ANSON, Director of Research, American Rolling Mills, Middletown, Ohio.

HEILPERIN, DR. M. A., c/o Institut Universitaire de Hautes Études Internationales, 132 rue de Lausanne, Genève, Switzerland.

HELLER, PROF. DR. WOLFGANG, Müegyetem-Rahpart 3, Budapest, Hungary.

HENDERSON, MR. A., King's College, Cambridge, England.

HENDERSON, DR. ROBERT, Crown Point, New York.

HENNEY, MR. HOMER J., Federal Crop Insurance Corporation, Department of Agriculture, Washington, D. C.

HERMANT, M. MAX, 10 rue Alfred Roll, Paris XVII, France.

HERVEY, DR. GEORGE W., 2347 South Meade Street, Arlington, Virginia.

HEYDEL, PROF. ADAM, Jagellonian University, Krakow, Poland.

HICKS, PROF. J. R., University of Manchester, Manchester, England.
HILL, MR. DANIEL A., 1633 Compton Road, Cleveland Heights, Ohio.
HIRSHFELD, MR. C. F., Detroit Edison Co., Detroit, Michigan.
HISATAKE, PROF. MASAO, Yokohama College, Rokkakubashi, Kanagawa, Yokohama, Japan.
HITCH, MR. CHARLES J., Queen's College, Oxford, England.
*HO, DR. FRANKLIN L., Institute of Economics, Nankai University, Tientsin, China.
HOTELLING, PROF. HAROLD, Columbia University, New York City.
HRASE, DR. JOSEF, Ovenecka, 23B, Praha XIX, Czecho-slovakia.
HUBER, DR. M., Director d'Études de l'Institut de Statistique, de l'Université de Paris, 173 boulevard Saint-Germain, Paris, France.
HUTCHINS, MR. JOHN G. B., 236 Grant Avenue, Highland Park, New Jersey.
HYTTEN, PROF. TORLIEV, c/o Economics Department, Bank of New South Wales, Box 2722, C.G.P.O., Sydney, New South Wales.

INSOLERA, PROF. FILADELFO, Via Velletri 10, Roma, Italy.
IONESCU SISESTI, PROF. G., l'Académie des Hautes Études Agricoles, Bucuresti, Rumania.
IVERSEN, DR. CARL, Professor Københavns Universitet, København, Denmark.

JACOBSON, MISS KATHARINE, 530 West 114th Street, New York City.
JACOBSSON, MR. PER., Economic Adviser to the Bank for International Settlements, Basle, Switzerland.
JAFFE, MR. HERMAN, Skogveien 12, Oslo, Norway.
JAFFÉ, PROF. WILLIAM, Department of Economics, Northwestern University, Evanston, Illinois.
JAHN, DIRECTOR GUNNAR, Det Statistiske Centralbyra, Oslo, Norway.
JAIN, DR. L. C., Professor of Economics, The University, Lahore, India.
JAMES, MR. WENDELL A., Farm Credit Administration, Washington, D.C.
JANKO, DR. JAROSLAV, Kolinska 12, Praha XII, Czecho-slovakia.
JANNACCONE, PROF. PASQUALE, 107 Corso Galileo Ferraris, Torino, Italy.
JANTZEN, MR. IVAR, Nationalbanken, København, Denmark.
JEMING, MR. JOSEPH, 1566 Longfellow Avenue, New York City.
JENKINS, MR. EDWARD, General Motors Corporation, 1775 Broadway, New York City
JENÖ, DR. ANDREICH, Müegyetemi m. tanar, Thököly ut 94, Budapest, Hungary.
JOHN, DR. ERNST, Dapontegasse 11/11, Wien III, Germany.
JORDAN, PROF. CHARLES, Maria-utca 46, Budapest VIII, Hungary.
JOSEPH, MISS MARGARET, 53 Linden Gardens, London, W.C.2, England.
JUDIK, DR. JÓZSEF, Ruszti ut 10, Budapest, Hungary.
JULIN, MR. A., 81 rue Louis Hap, Bruxelles, Belgium.
JUTILA, PROF. DR. K. T., Bank of Finland, Helsinki, Finland.

KAFURI, DR. JORGE FELIPPE, Professor Catedratico da Escola Nacional de Engenharia, Universidade do Brasil, Rio de Janeiro, Brazil.
KALDOR, DR. NICHOLAS, London School of Economics, Houghton Street, Aldwych, London, England.
KALECKI, MR. MICHAL, 42 Garden Walk, Cambridge, England.
KARRER, DR. S., Director of Research, Consolidated Gas, Electric Light & Power Company, Baltimore, Maryland.
KARVAS, DR. IMRICH, A., Kapitulská, Bratislava, Slovakia.
KAUFMANN, PROF. DR. FELIX, 533 W. 232nd Street, New York City.

KAUL, DR. BHAGIRATH NATH, Financial Section, League of Nations, Genève, Switzerland.

KEILHAU, PROF. WILHELM, Pilestredet 94, Oslo, Norway.

KELLER, DR. HENRY, JR., Dept. of Agricultural Economics, Rutgers University, New Brunswick, New Jersey.

KELLOGG, PROF. LESTER, College of Commerce and Administration, Ohio State University, Columbus, Ohio.

KELLY, DEAN E. J., Santa Clara University, Santa Clara, California.

KEMÉNY, DR. DEZSO, Szemere-Utca 6, Budapest V, Hungary.

KENÉZ, DR. BÉLA, Tudományegyetem, Budapest, IV, Hungary.

KEYNES, MR. JOHN M., King's College, Cambridge, England.

KING, MR. FREDERICK, 609 West 115th Street, New York City.

KING, MR. HAROLD J., Department of Accounting, School of Business Administration, The University of Pittsburgh, Pittsburgh, Pennsylvania.

KING, MR. K. P., Research Department, Central Bank of China, 15 The Bund, Shanghai, China.

KING, PROF. W. I., School of Commerce, New York University, Washington Square East, New York City.

KIRSCHEN, MR. ÉTIENNE-SADI, 32 rue aux Laines, Bruxelles, Belgium.

KLEEREKOPER, DR. S., Johannes Verhulststraat 22, Amsterdam Z, The Netherlands.

KLONOV, DR. VLADIMÍR, Institute of Agricultural Economics, Sadová, Praha-Dejvice, Czecho-slovakia.

KNAUTH, MR. OSWALD, 366 Fifth Avenue, New York City.

KNUDSEN, MR. ARNE, "Brage," Prinsensgt. 22, Oslo, Norway.

KOLODZIEJCZYK, MR. STANISLAW, Statistical Laboratory, S.G.G.W., Rakowiecka 8, Warszawa, Poland.

KOOPMANS, PROF. J. G., Sweelinckplein 29, 's Gravenhage, The Netherlands.

KOOPMANS, DR. T., League of Nations, Genève, Switzerland.

KOVERO, DR. ILM, Professor of Public Finance, University of Helsinki, Helsinki, Finland.

KOZELKA, DR. R. L., University of Minnesota, Minneapolis, Minnesota.

KREPS, DR. T. J., Stanford University, California.

KROMPHARDT, PROF. W., Universität-Johannisstr. 9, Muenster-Westf., Germany.

KÜHNE, DR. OTTO, Kurfürstendamm 113, Berlin-Hallensee, Germany.

KULL, DR. WALTER, International Labour Office, Genève, Switzerland.

KURIMURA, DR. YUKICHI, Faculty of Law and Letters, Kyushu Imperial University, Fukuoka, Japan.

KUZNETS, DR. SIMON, 2 Bent Road, Bowling Green, Media, Pennsylvania.

LANDAU, MR. LUDWIK, Kosaykowa 51m.42, Warszawa I, Poland.

LANDRY, M. ADOLPHE, 10 avenue du Square, Paris XVI, France.

LANGE, MR. CARVEL, Institute of Applied Econometrics, 405 Lexington Avenue, New York City.

LANGE, DR. OSKAR, Department of Economics, University of Chicago, Chicago, Illinois.

LANGE-NEILSEN, MR. FR., The State Pension Fund, Keysersgt. 8, Oslo, Norway.

LANGMUIR, MR. DEAN, 90 Broad Street, New York City.

LASORSA, PROF. GIOVANNI, Via Padova, Roma, Italy.

LATSHAW, DR. V. V., Department of Mathematics, Lehigh University, Bethlehem, Pennsylvania.

LEAVENS, MR. DICKSON H., Cowles Commission for Research in Economics, The University of Chicago, Chicago, Illinois.

LE CORBEILLER, DR. PH., 278 boulevard Raspail, Paris, France.
LEDERER, DR. WALTER, Department of Economics, Queens College, Flushing, New York.
LEDUC, DR. GASTON, 51 boulevard Beauséjour, Paris XVI, France.
LEGUIZAMON, SR. GUILLERMO E., Chairman of Local Boards, Southern & Western Railways, Paseo Colon 185, 7th Floor, Buenos Aires, Argentina.
LEONTIEF, PROF. WASSILY, 37 Holyoke House, Harvard University, Cambridge, Mass.
LERNER, MR. A. P., c/o Rockefeller Foundation, 49 West 49th St., New York City
LEVY, PROF. PAUL, 80 Avenue des Rogations, Bruxelles, Belgium.
LEWIS, MR. HAROLD GREGG, 1536 East 60th Street, Chicago, Illinois.
LEWY, MR. STEFAN, Czerwonego Krzyza 15, Warszawa, Poland.
LIEU, DR. D. K., c/o The China Institute of Economic and Statistical Research, P.O. Box No. 4019, Shanghai (18), China.
LIMANOWSKI, PROF. ZYGMUNT, Rakowiecka 6, Warszawa, Poland.
LIMPERG, PROF. TH., JR., De Lairessestraat 96, Amsterdam Z, The Netherlands.
LINDAHL, PROF. ERIK, Pedellgatan 15, Lund, Sweden.
LINDER, DR. ARTHUR, Sulgenauweg 20, Bern, Switzerland.
LIPINSKI, PROF. EDWARD, Olimpijska 39, Warszawa XII, Poland.
LIPMAN, MR. F. L., Wells Fargo Bank and Union Trust Company, San Francisco, California.
LIVI, PROF. LIVIO, R. Istituto Superiore di Scienze Sociali e Politiche, Via Laura, 48, Firenze, Italy.
LOESCH, DR. AUGUST, Heidenheim (Wuerrt.), Germany.
LOISY, MR. TH., Villa Les Paquerettes, Rue Paul de Rouville, Montpellier (Hérault), France.
LORENZ, MR. M. O., 3510 Porter Street N. W., Washington, D. C.
LORGE, DR. IRVING, Teachers College, Columbia University, New York City.
LORIA, PROF. A., Corso Oporto 26, Torino, Italy.
LOTHERINGTON, MR. JOHN, Statens Pensjonskasse, Oslo, Norway.
LOTKA, DR. ALFRED J., Metropolitan Life Insurance Co., New York City.
LOVASY, DR. GERTRUDE, 3 Willoughby Road, London N.W. 3, England.
LOVEDAY, MR. ALEX, Directeur de la section financière de la Societé des Nations, Genève, Switzerland.
LÖWE, DR. ADOLF, 10, Wilbraham Road, Manchester, England.
LUTFALLA, M. GEORGES, 40, rue de la Voie Verte, Paris XIV, France.
LUZZATTO-FEGIZ, PROF. PIERPAOLO, 8 Via G. Revere, Trieste, Italy.

MACDOUGALL, MR. GEORGE D. A., Department of Economics, The University, Leeds, England.
MACHLUP, DR. FRITZ, Dept. of Economics, University of Buffalo, Buffalo, N. Y.
MCINTYRE, PROF. FRANCIS E., Dept. of Economics, Stanford University, California.
MCLEAN, MR. W., c/o University Buildings, Marino, Madras, India.
MADHAVA, PROF. K. B., University of Mysore, Mysore, India.
MADOW, DR. W. G., 7 West 14th Street, New York City.
MAKOWER, MISS HELEN, Institute of Statistics, 74 High Street, Oxford, England.
MALKARY, MR. HENRY J., 9 Polo Road, Great Neck, Long Island, N. Y.
MANOILEJO, M. MIHAIL, Professeur d'Économie Politique, l'École Polytechnique, Rue Mussolini 27, Bucuresti, Rumania.
MARGET, PROF. ARTHUR W., School of Business Administration, University of Minnesota, Minneapolis, Minnesota.
MARLIO, M. LOUIS, 23 rue Balzac, Paris VIII, France.

MARN, MR. L. MATTSON, Söderhamm, Sweden.
MARSCHAK, DR. JAKOB, All Souls College, Oxford, England.
MARTIN, MR. ANTONIO DE MIGUEL, Chef des Statistiques Economiques, Service National de Statistique, Ministere d'Organisations, Santander, Spain.
MARTIN, DR. FRANZ, Porzellangasse 16, Wien IX, Germany.
MARTZLOFF, PROF. GEORGES, 8 rue Bertron, Sceaux près Paris (Seine), France.
MAS, DR. GONSAGUE, Roujan (Hérault), France.
MASCI, PROF. GUGLIELMO, Facoltà di Giurisprudenza, R. Università, Città Universitaria, Roma, Italy.
MASNATA, DR. ALBERT, Post Box 4, Lausanne, Switzerland.
MAVERICK, DR. LEWIS A., University of California at Los Angeles, 405 Hilgard Ave., Los Angeles, California.
MAYER, PROF. HANS, Universität, Wien, Germany.
MAYER, DR. JOSEPH, Library of Congress, Washington, D.C.
MAYER, MR. LEO KENNETH, 215 Middle Neck Road, Great Neck, Long Island, N. Y.
MAZURE, MR. JANNIS PIETER, 38 Houtrustlaan, 's Gravenhage, The Netherlands.
MEANS, MR. GARDINER C., Box 43, Route 1, Vienna, Virginia.
MEARS, PROF. ELIOT GRINNELL, 593 Gerone Road, Stanford University, California.
*MEDICI, DR. GIUSEPPE, R. Istituto Superiore Agrario di Perugia, Perugia, Italy.
MEDOLAGHI, PROF. PAOLO, R. Università, Via Terme Deciane (Aventino) 7, Roma, Italy.
MELLER, MR. WILLIAM, Ken-u 8, Budapest, Hungary.
MENDERSHAUSEN, DR. HORST, Colorado College, Colorado Springs, Colorado.
MENGARINI, PROF. PUBLIO, University of Naples, Rione Sirignane 9, Napoli, Italy.
MENGER, PROF. KARL, Box 206, Notre Dame, Indiana.
MIDUTANI, PROF. K., University of Commerce, Kobe, Japan.
MILHAU, PROF. JULES, Rue Blanchard, Agde (Hérault), France.
MILLIKAN, MR. MAX, 1285 Boulevard, Apt. 46, New Haven, Connecticut.
MILLS, PROF. FREDERICK C., School of Business, Columbia University, New York City.
MINOST, MR. ÉMILE, Administrateur-Directeur Générale du Crédit Foncier Égyptien, Cairo, Egypt.
MISES, PROF. DR. LUDWIG, 3 Avenue Dumas, Genève, Switzerland.
MISES, PROF. RICHARD DE, Pierce Hall, Harvard University, Cambridge, Massachusetts.
MITCHELL, PROF. WESLEY C., 2 Horatio Street, New York City.
MITNITZKY, DR. MARK, 19 rue Copernic, Paris XVI, France.
MOLINA, MR. E. C., Switching Theory Engineer, Bell Telephone Laboratories, 463 West Street, New York, N. Y.
MOLINARI, DR. ALESSANDRO, 32 Viale Liegi, Roma 1/12, Italy.
MONTGOMERY, MR. J. K., Institut International d'Agriculture, Roma, Italy.
MOONEY, MR. JAMES D., General Motors Corporation, 1775 Broadway, New York City.
MOORE, PROF. H. L., Cornwall, New York.
MORET, DR. JACQUES, 19 rue de la Convention, Paris XV, France.
MORGENSTERN, DR. OSKAR, Princeton University, Princeton, New Jersey.
MORICE, PROF. EUGENE, 9 rue G. Clemenceau, Sannois (Seine et Oise), France.
MORK, DOCENT R., Landbrukshoiskolen, As, Norway.
MORRISON, MR. NATHAN, 450 Ocean Parkway, Brooklyn, N. Y.
MORTARA, PROF. GIORGIO, Rua Raul Pompeia 109 (Copacabona), Rio de Janeiro, Brazil.
MORTON, DR. J. E., Dept. of Economics, Knox College, Galesburg, Illinois.

MOURRÉ, BARON CHARLES, 5 rue du Boccador, Paris VIII, France.
MOWBRAY, PROF. A. H., 119 South Hall, University of California, Berkeley, California.
MUDGETT, PROF. BRUCE D., University of Minnesota, Minneapolis, Minn.
MYRDAL, PROF. GUNNAR, Norrtullsgatan 2, Stockholm, Sweden.

NAKAYAMA, PROF. ICHIRO, Commercial University of Tokyo, Tokyo, Japan.
NAVRATIL, PROF. DR. ÁKOS, Apponyi tér 1, Budapest IV, Hungary.
NEIFELD, DR. MORRIS R., 38 Mayhew Drive, South Orange, New Jersey.
NEISSER, DR. HANS, 227 Kenyon Avenue, Swarthmore, Pennsylvania.
NEUBAUER, DR. JULIUS, Iskola utca 13, Budapest II, Hungary.
NEYMAN, DR. J., Department of Mathematics, University of California, Berkeley, California.
NICHOL, PROF. A. J., Department of Economics, University of California, Berkeley, California.
NIXON, MR. JAMES WILLIAM, International Labour Office, Genève, Switzerland.
NØLKE, MR. ALFRED, Parkveien 62, Oslo, Norway.
NORRIS, DR. NILAN, 2800 University Avenue, New York City.
NORTON, DR. J. PEASE, 460 Ocean Avenue, West Haven, Conn.
NOYES, MR. C. REINOLD, 455 East 51st Street, New York City.
NYBOLLE, DR. HANS C., Hostrups Have 2, København, Denmark.

O'BRIEN, MR. P. K., "Bellus," Oaklands, Welwyn, Herts, England.
*ODATE, MR. YOSHIO, The Nippon Life Assurance Company, 7, Imabashi Shichome, Osaka, Japan.
OHLIN, PROF. BERTIL, Handelshögskolan, Stockholm, Sweden.
OLSHEVSKY, MR. LOUIS, 1317 Prospect Avenue, Bronx, New York City.
ONICESCU, PROF. OCTAV, Strada Rozeloz 9, Bucuresti, Rumania.
OPIE, DR. REDVERS, Magdalen College, Oxford, England.
OPPEGAARD, MR. K. F., Munkedams Vn. 35, Oslo, Norway.
ORE, PROF. OYSTEIN, Yale University, New Haven, Conn.
ORLOFF, DR. M. SCHELLENBERG, Calle 18 de Julio 1333, Montevideo, Uruguay.
ORLOW, MR. WLADIMIR, 194 Chaussée d'Ixelles, Bruxelles, Belgium.

PABST, DR. WILLIAM R., Department of Economics, Tulane University, New Orleans, Louisiana.
PALMSTROM, HRR. HENRIK, Livsforsikringsselskapet Brage, Oslo, Norway.
PANKRAZ, DR. OTOMAR, Charles University, U Karlova 3, Praha II, Czechoslovakia.
PAPI, PROF. UGO, Via Plinio 7, Roma, Italy.
PARADISO, DR. L. J., 1717 R Street, N.W., Washington, D. C.
PAYNE, DR. CHARLES K., Washington-Square College, New York, N. Y.
PEARSON, MR. EGON S., Senior Lecturer, University of London, University College, Gower Street, London, W.C. 1, England.
PEARSON, PROF. F. A., Cornell University, Ithaca, New York.
PEDERSEN, DR. JORGEN, Institute of Economics, University of Aarhus, Aarhus, Denmark.
PEDERSEN, PROF. P. O., Skovalle 85, Bagsvaerd, Denmark.
PERLO, MR. VICTOR, Federal Home Loan Bank Board, Washington, D.C.
PETER, DR. HANS, Beuckestrasse 12, Berlin-Zehlendorf, Germany.
PETERSEN, MR. O. STRANGE, Stadion Alle 32, Aarhus, Denmark.
PETRICSKO, PROF. NIKOLAUS, Vigado-tér 1, Penzintezeti Kozpont, Budapest IV, Hungary.

PICARD, M. ÉMILE, Secrétaire perpetuel de l'Academie des Sciences, 25 quai Conti, Paris, France.

PIEKALKIEWICZ, M. JEAN, Czernioków Miasto Ogród, Goraszewska 4, Warszawa, Poland.

PIETRA, PROF. COMM. GAETANO, Regia Università, Padova, Italy.

PIXLEY, DR. HENRY, Wayne University, Detroit, Michigan.

POLAK, DR. J. J., League of Nations, Genève, Switzerland.

*POSE, M. ALFRED, Le Directeur Général, Banque Nationale pour le Commerce et l'Industrie, Paris, France.

POSTHUMA, MR. S., Terhoffsteedeweg 6, Overeen, The Netherlands.

POTRON, PROF. MAURICE, Institut Catholique de Paris, 42 rue de Grenelle, Paris VI, France.

POULSEN, MR. TORBJØRN H., Universitetetes Socialøkonomiske Institutt, Oslo, Norway.

PREINREICH, DR. GABRIEL A. D., 17 East 42nd Street, New York City.

PREISSER, DR. ERICH, Paulstr. 48, Rostock, Germany.

PRIBRAM, PROF. DR. KARL, 3625 16th Street, N. W., Washington, D. C.

PRICE, DR. IRENE, Department of Mathematics, State Teachers' College, Oshkosh, Wisconsin.

PULLERITS, MR. ALBERT, Central Bureau of Statistics, Tallinn, Estonia.

QVALE, MR. PAUL, Havaristalistikkontoret, Fr. Nansensplass 5, Oslo, Norway.

RADICE, DR. E. A., League of Nations, Genève, Switzerland.

RASTOKIN, DR. ING. PETER D., Zemědělskaj ústav Učetnicko-Spravovědny, Praha, XIX, Czecho-slovakia.

RAUTENSTRAUCH, DR. WALTER, Department of Industrial Engineering, Columbia University, New York City.

RAWLES, DEAN THOMAS H., Colorado College, Colorado Springs, Colorado.

RAYMOND, PROF. FAIRFIELD, 28 Meadow Way, Cambridge, Massachusetts.

REITSMA, IR. J. M. I., Krabbenbosweg, 120, Hengelo (O), The Netherlands.

RHODES, DR. E. C., London School of Economics, Houghton Street, London, W. C. 2, England.

RICCI, PROF. UMBERTO, Faculty of Law, Giza, Egypt.

RICHARDSON, MR. BURDICK, Lane Building, Davenport, Iowa.

RICHTER-ALTSCHAFFER, DR. HANS, 1 Grosvenor Square, Room 204, London W. 1, England.

RIDER, PROF. PAUL R., Washington University, St. Louis, Missouri.

RIETZ, PROF. HENRY LEWIS, Physics Building, University of Iowa, Iowa City, Iowa.

RIPLEY, MR. JULIEN A., JR., P. O. Box 591, Alexandria, Virginia.

RIST, PROF. CHARLES, 18 bis rue du Parc de Clagny, Versailles, France.

ROA, MR. EMETERIO, Actuary, Insular Life Assurance Co., Ltd., P. O. Box 128, Manila, Philippine Islands.

ROBBINS, PROF. LIONEL C., London School of Economics, Houghton St., Aldwych, London, W.C.2, England.

ROBERTS, MR. MELVIN J., Colorado National Bank of Denver, Denver, Colorado.

ROBINSON, MR. H. W., 183 Iffley Road, Oxford, England.

ROLL, PROF. ERICH, 29 Hallgate, Cottingham, East Yorkshire, England.

ROMBOUTS, ING. ALBERT LOUIS GERARD MARIE, 2 de Emmastraat 276 's Gravenhage, The Netherlands.

ROMIG, MR. HARRY G., Bell Telephone Laboratories, 463 West Street, New York City.

ROODKOWSKY, MR. NIKITA D., National Arts Club, 15 Gramercy Park, New York City.

ROOS, DR. CHARLES F., Institute of Applied Econometrics, 405 Lexington Avenue, New York City.

ROSENBORG, DR. A., League of Nations, Genève, Switzerland.

ROSSI, PROF. LIONELLO, Via Amalasunta 7, Roma, Italy.

ROUQUET, ING. VICTOR, Professeur adjoint au College, Clermont l'Hérault, France.

ROUSSEAUX, MR. PAUL, Institut des Recherches Économiques, Bibliothèque, Place du Peuple, Louvain, Belgium.

ROY, PROF. RENÉ, 7 rue de la Cité universitaire, Paris XIV, France.

ROYOT, M. GEORGES, Secretary of the Banking Department of the Bank of International Settlements, Basle, Switzerland.

RUEFF, M. JACQUES, 51 rue de Varenne, Paris VII, France.

RUNGE, PROF. LULU L., University of Nebraska, Lincoln, Nebraska.

RUTSKI, DR. J., Mostowa 3a-20, Wilno, Poland.

SACHS, DR. ALEXANDER, Lehman Corporation, 1-3 South William Street, New York City.

SAMUELSON, MR. PAUL A., 13A Ware Street, Cambridge, Massachusetts.

SANKPAL, PROF. G. M., Sydenham College of Commerce, Bombay, India.

SARGENT, MR. NOEL, 14 West 49th Street, New York City.

SASULY, MR. MAX, 2800 Ontario Road, N. W., Washington, D. C.

SAWKINS, MR. D. T., Dept. of Statistics, The University, Sydney, Australia.

SCATES, DR. DOUGLAS, College Station, North Carolina.

SCHARFF, MR. MAURICE R., 285 Madison Avenue, New York City.

SCHIFF, DR. ERICH, 429 S. Division Street, Ann Arbor, Michigan.

SCHNEIDER, PROF. DR. ERICH, University of Aarhus, Aarhus, Denmark.

SCHOENBAUM, DR. EMIL, Professeur a l'Université, Praha, Czecho-slovakia.

SCHULTZ, PROF. T. W., Department of Economics, Iowa State College, Ames, Iowa.

SCHUMACH, MR. G. J. D., 74 Outwoods Drive, Loughborough, Leicestershire, England.

SCHUMPETER, PROF. JOSEPH A., 7 Acacia Street, Cambridge, Massachusetts.

SCHWARZ, ING. FRANKTISĚK, Přemyslova 245, Praha-Podoli, Czecho-slovakia.

SCOTT, MR. HAROLD CLAUDE, 78 Mayfield Road, Sanderstead, Surrey, England.

SCOVILLE, MR. JOHN W., Chief Statistician, Chrysler Corporation, Detroit, Michigan.

SECRIST, PROF. HORACE, Bureau of Business Research, Northwestern University School of Commerce, McKinlock Campus, Chicago, Illinois.

SHACKLE, MR. G. L. S., Dept. of Economics, The University, St. Andrews, Fife, Scotland.

SHENG, MR. T., c/o Cheng Fu & Co., P.O. Box No. 1399, Shanghai, China.

SHEWHART, DR. W. A., Bell Telephone Laboratories, 463 West Street, New York City.

SHOHAT, PROF. J., Mathematics Department, University of Pennsylvania, Philadelphia, Pennsylvania.

SIBATA, MR. KEI, Faculty of Economics, Kyoto Imperial University, Kyoto, Japan.

SIMMONS, MR. FREDERIC L., G.M.-P. Murphy & Co., 37 Threadneedle Street, London, E.C.2, England.

SINDING, DR. THOMAS, Economist, Gabelsgt. 23, Oslo, Norway.

SIPOS, DR. ALEXANDER, Zsigmond-u. 20, Budapest II, Hungary.

SKAUG, MR. ARNE, Central Bureau of Statistics, Oslo, Norway.

SKRZYWAN, MR. WACLAW, Polish Institute for Economic Research, Elektoralna 2, Warszawa, Poland.

SMITH, DR. E. DILLON, 1406 S. First Avenue, Arcadia, California.

SMITH, MR. EDGAR W., General Motors Corporation, 1775 Broadway, New York City.

SMITH, MR. H., Ruskin College, Oxford, England.

SMOLENSKY, DR. PIETRO, Gerente La Continental, Diagonal Norte 555, Buenos Aires, Argentina.

SMUTNY, DR. PAVEL, State Statistical Office, Bělského, 2, Praha, VII, Czechoslovakia.

SOKOLOVE, MR. HENRI, 1343 Connecticut Avenue, Washington, D. C.

SOUTO, DR. JOSE BARRAL, Professor of Statistics, University of Buenos Aires, Buenos Aires, Argentina.

SPENGLER, MR. J. J., Duke University, Durham, North Carolina.

SPIETHOFF, PROF. ARTHUR, Pappelsdorfer Alle 25a, Bonn, Germany.

SRAFFA, PROF. PIERO, King's College, Cambridge, England.

STACKELBERG, DR. HEINRICH BARON VON, Unter den Linden 76, Berlin-Dahlem, Germany.

STAEHLE, DR. HANS, c/o Rockefeller Foundation, 49 West 49th Street, New York City.

STAHL, MR. CAREL, JR., Willemskade 4, Rotterdam, The Netherlands.

STAMP, THE LORD, G. C. B., C. B. E., Park Hill Road, Shortlands, Kent, England.

STAUBER, MR. B. R., 4926 Eskridge Terrace, N.W., Washington, D. C.

STAUDE, DR. HEINRICH, Schnorrstrasse 26, Leipzig W. 31, Germany.

STEFANOFF, PROF. IVAN, Professor of Statistics, Commercial University, Ulitza Tzarska 1, Svishtov, Bulgaria.

STEFFENSON, PROF. DR. J. F., Ehlersvej 8, Hellerup, Denmark.

STEINDL, DR. JOSEPH, Balliol College, Oxford, England.

STEPHAN, MR. FREDERICK F., American Statistical Association, Room 210, Normandy Building, 1626 K. St., N.W., Washington, D.C.

STEVENSON, DEAN R. A., School of Business Administration, University of Minnesota, Minneapolis, Minnesota.

STEWART, DR. W. BLAIR, 4801 Brandywine Street, N.W., Washington, D.C.

STJERNQVIST, MR. HENRY, Det Statistisk Department, København, Denmark.

STONE, MR. JOHN RICHARD, N. Chelsea Studios, 416 Fulham Road, London, S.W. 6, England.

STRIGL, DR. RICHARD, Schulgasse 90, Wien XVIII, Germany.

SUGIMOTO, PROF. EIITI, Tokyo University of Commerce, Tokyo, Japan.

SUN, DR. C., c/o Mr. C. S. Yao, Kincheng Banking Corporation, 200 Kiangse Road, Shanghai, China.

SURÁNYI-UNGER, DR. THEO, Francis Joseph University, Szeged, Hungary.

SURVEYER, DR. ARTHUR, International Bond and Share Corporation, Suite 1004, Dominion Square Building, 1010 St. Catherine Street West, Montreal, Canada.

SVENNILSON, MR. INGVAR, Stockholms Socialvetenskapliga Institut, Odengatan 61, Stockholm, Sweden.

SWIANIEWICZ, DR. STANISLAW, Docent, Wilno Universytet, Wilno, Poland.

SZTURM DE SZTREM, MR. EDWARD, Wawelska, 78-17, Warszawa, Poland.

TAITEL, MR. MARTIN, 4615 Chestnut Street, Bethesda, Maryland.

TAKATA, PROF. YASUMA, Faculty of Economics, Kyoto Imperial University, Kyoto, Japan.

TAO, DR. L. K., Director, Institute of Social Sciences, c/o Institute of History and Philology, Kunming, Yunnan, China.

TASSINARI, PROF. GIUSEPPE, Via Lambro 2, Roma, Italy.

THEISS, DR. EDWARD, Ballagi Mor. u 12, Budapest XI, Hungary.

THESEN, MR. GUDBRAND, Cond. Actuar, Idun, Kirkegaten 24, Oslo, Norway.

THIRRING, DR. LOUIS, Zsigmond-utca 1, Budapest, Hungary.

THOMAS, MR. CARL, 43 Holyoke House, Cambridge, Mass.

THOMPSON, DR. JOHN MURRAY, c/o Western Division of Agricultural Adjustment Administration, U. S. Department of Agriculture, Washington, D. C.

THORBJÖRNSEN, MR. CHRISTIAN, Aktuar, Rikstrygdeverket, Oslo, Norway.

THOREN, MISS HERDIS, Livsforsikringsselskapet Brage, Prinsensgt. 22, Oslo, Norway.

THURMAN-MOE, MR. J., De Norske Livsforsikringsselskapers Statistiske Kontor, Stortingsgaten 22, Oslo, Norway.

TIMOSHENKO, DR. V. P., Food Research Institute, Stanford University, California.

*TIMPE, PROF., Dianastrasse 21, Berlin-Schlachtensee, Germany.

TINBERGEN, DR. J., Kauwlaan 4, 's Gravenhage, The Netherlands.

TINTNER, PROF. GERHARD, Dept. of Economics, Iowa State College, Ames, Iowa.

TOBATA, PROF. SEIICHI, Faculty of Agriculture, Tokyo Imperial University, Tokyo, Japan.

TOBILCOCK, MR. D. H., c/o Govt. Statistical Bureau, P. O. Box 791, Salisbury, S. Rhodesia, S. Africa.

TOKOYAMA, PROF. TSUNESABURO, Department of Politics and Economics, Waseda University, Tokyo, Japan.

TOLLEY, PROF. HOWARD R., Room 3095, South Building, Dept. of Agriculture, Washington, D. C.

TOMLINSON, PROF. DR. F. R., University of Pretoria, Pretoria, South Africa.

TOUT, MR. HERBERT, 10 Royal Park, Clifton, Bristol 8, England.

TREVOUX, PROF. F., Faculté de Droit de Lyon, Lyon, France.

TRUKSA, DR. LADISLAV, Praha XI-1800, Czecho-slovakia.

TUTTLE, DR. CHARLES R. M., 1831 Jefferson Place, N.W., Washington, D. C.

UGARTE, MR. SALVADOR, Calle V. Carranza No. 42, Mexico, D. F., Mexico.

UGGE, PROF. ALBINO, Via Carlo Farini N. 55, Milano, Italy.

ULLIAN, MRS. FRIEDA S., Research Fellow, Radcliffe College, Cambridge, Massachusetts.

ULLMO, MR. JEAN, Répétiteur d'analyse a l'École Polytechnique, 113 rue de la Tour, Paris XVI, France.

VALK, DR. WILLIAM L., Weissenbruchstraat, 398, 's Gravenhage, The Netherlands.

VAN DE GINSTE, MR. FERNAND ARTHUR, Agent Territorial Principal, Masi Manimba, Province Leopoldville, Belgian Congo.

VANDELLOS, DR. JOSÉ A., Direccion de Estadistica, Salvador de Leon a Socarras No. 44, Caracas, Venezuela.

VAN DER MEER, MR. ARY PIETER, Prins. Bernhardkade 9, Hilligersberg, The Netherlands.

VAN DER MEER, MR. BARELD, Kanaalweg 125, Utrecht, The Netherlands.

VAN DER WIJK, MR. J., Corn. Krusemanstr. 27II, Amsterdam Z, The Netherlands.

VAN DORP, DR. E. C., Villa Jamga, Bloemendaal, The Netherlands.

VAN GENECHTEN, DR. R., Willemsplantsoen 10, Utrecht, The Netherlands.

VAN GOGH, MR. VINCENT WILLEM, 209 Heerengracht, Amsterdam, The Netherlands.

VAN PRAAG, DR. IR. M. M., 49 Nassaulaan, Schiedam, The Netherlands.

VAN WINKLE, PROF. E. H., Rensselaer Polytechnic Institute, Troy, New York.

VARGA, DR. STEPHEN, Director, Hungarian Institute for Economic Research, 8, Alkotmány-utca, Budapest, Hungary.

VERGARA, SR. ROBERTO, Av. Manuel Montt. 1646, Santiago, Chile.

VERRYN-STUART, PROF. G. M., Emmalaan 6, Utrecht, The Netherlands.

VIAL, DR. EDMUND E., Bureau of Agricultural Economics, Washington, D. C.

VICKERY, DR. CHARLES W., Chief Statistician, Highway Planning Survey, 304 E. 5th Street, Austin, Texas.
VIDAS, DR. PIERO, Via S. Basilio 14, Roma, Italy.
VINCI, PROF. FELICE, Viale Aldini 3, Bologna, Italy.
VINOGRADOFF, MR. D. I., Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pennsylvania.
*VLEUGELS, PROF. DR. WILHELM, Argelanderstrasse 12, Bonn am Rhein, Germany.
VOLTERRA, SENATOR VITO, 17 Via in Lucina, Roma, Italy.
VON BECKERATH, PROF. ERWIN, Universität Köln, Köln, Germany.
VON DER NAHMER, MR. ROBERT K., 12 Roeklaan, Eindhoven, The Netherlands.
VON MERING, BARON OTTO, 110 Irving Street, Cambridge, Massachusetts.
VON NEUMANN, PROF. JOHN, 65 Stockton Street, Princeton, New Jersey.
VON SZELISKI, MR. VICTOR S., 4408 Chrysler Building, 405 Lexington Avenue, New York City.

WAITE, PROF. WARREN C., University Farm, St. Paul, Minnesota.
WAFFENSCHMIDT, PROF. W., Bismarckstrasse 59, Karlsruhe, Germany.
WAGEMANN, PROF. DR. E., Fasanenstrasse 6, Berlin-Charlottenburg 2, Germany.
WAKAR, DR. ALEKSY, Rakowiecka 43A, Warszawa, Poland.
WALD, DR. A., Fayerweather Hall, Columbia University, New York City.
WALKER, PROF. HELEN M., Teachers College, Columbia University, New York, N. Y.
WARDWELL, PROF. CHARLES A. R., Department of Economics, Northwestern University, Evanston, Illinois.
WAUGH, DR. FREDERICK V., U. S. Bureau of Agricultural Economics, Washington, D. C.
WEAVER, MR. DONALD, 1332 Vermont Ave., N. W., Washington, D. C.
WEDERVANG, PROF. INGVAR, Norges Handelshøiskole, Bergen, Norway.
WEIDA, PROF. FRANK M., The George Washington University, Washington, D. C.
WEISZMANDEL, MISS FRANCES E., 33-25 72nd St., Jackson Heights, New York City.
WELCH, MR. B. L., 10 Holmlands Park South, Sunderland, England.
WELLS, PRESIDENT HERMAN B., Indiana University, Bloomington, Indiana.
WENZLICK, MR. ROY, Real Estate Analysts, Inc., St. Louis, Missouri.
WESTERFIELD, PROF. R. B., Yale University, New Haven, Conn.
WHITMAN, DR. ROSWELL H., Office of the Controller, R. H. Macy & Co., New York City.
WICKSELL, PROF. SVEN DAG, University of Lund, Lund, Sweden.
WILKS, PROF. S. S., Fine Hall, Princeton University, Princeton, New Jersey.
WILLIAMS, DR. JOHN H., 148 Coolidge Hill, Cambridge, Massachusetts.
WILSON, PROF. E. B., School of Public Health, Harvard University, Shattuck Street, Boston 17, Mass.
WILSON, MISS ELIZABETH, 1 Waterhouse Street, Cambridge, Massachusetts.
WINKLER, PROF. DR. WILHELM, Ghelengasse 30, Wien XIII, Germany.
WISNIEWSKI, PROF. JAN, Rakowiecka 6, Warszawa, Poland.
WOLD, DR. HERMAN O. A., University, Stockholm, Sweden.
WOLF, MR. A. F., Hills Brothers Company, 110 Washington Street, New York City
WOLFF, M. ROBERT, 116 Avenue des Nations, Bruxelles, Belgium.
WOLFOWITZ, MR. JACOB, 308 Cumberland Street, Brooklyn, New York.
WOLLERT, MR. BERTIL, Lokevägen 12, Djursholm, Sweden.
WOODBURY, DR. ROBERT M., c/o International Labour Office, Genève, Switzerland.
WOODYARD, DR. ELLA, Teachers College, Columbia University, New York City.

WORKING, PROF. E. J., Department of Agricultural Economics, University of Illinois, Urbana, Illinois.

WORKING, PROF. HOLBROOK, Food Research Institute, Stanford University, California.

WOYTINSKY, MR. WLADIMIR, 4943 Butterworth Place, N. W., Washington, D. C.

WRIGHT, MR. CARL MAJOR, Member of Economic Section, International Labour Office, Genève, Switzerland.

WU, MR. TAYEH, Directorate of Statistics, National Government, Chungking, China.

WUBNIG, MR. ARTHUR, Railroad Retirement Board, Washington, D.C.

YAMADA, PROF. ISAMU, Yokohama College, Rokkakubashi, Kanagawa, Yokohama, Japan.

YNTEMA, PROF. THEODORE O., Cowles Commission for Research in Economics, The University of Chicago, Chicago, Illinois.

ZAGOROFF, DR. SLAVCHO, Parchevich 36, Sofia, Bulgaria.

ZAGORSKI, MR. JOZEF, Instytut Konjunktur i cen, Al. Jerozolimskie 85, m. 3, Warszawa 1, Poland.

ZASSENHAUS, DR. HERBERT, 1703 Washington Street, Huntingdon, Pennsylvania.

ZAYCOFF, DR. RASHCO, Statistical Institute for Economic Research, State University, Shipka 6, Sofia IV, Bulgaria.

ZEMPEL, PROF. ARNOLD, Washington University, St. Louis, Missouri.

ZEUTHEN, DR. F., Hostrups, Have 1, København V, Denmark.

ZOLL, MR. WALTER, Lindelshang 5, Wurzburg, Germany.

ZUJOVIC, PROF. MILAN J., Prof. à la Faculté de Droit, Kralja Milana 42, Beograd, Yugoslavia.

ZWEIG, DR. FERDYNAND, University of Kraków, Lobzowska 4, Kraków, Poland.

SUMMARY

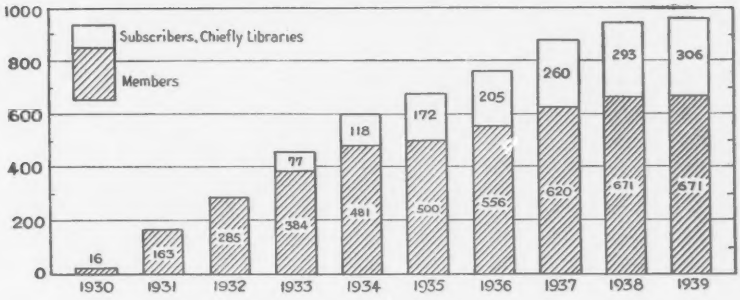
Members, October 1, 1938		671
New Members Added		47
		718
Less: Deaths	8	
Resignations	36	
Dropped for nonpayment of dues	3	47
Members, October 1, 1939		671

OBITUARY

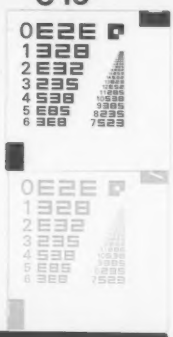
ECONOMETRICA records with deep regret that the deaths of the following members of the Econometric Society have occurred during the past year.

PROFESSOR EDGARD ALLIX
PROFESSOR C. COLSON
DR. ANGELO DELLA-RICCIA
PROFESSOR EMIL LEDERER
PROFESSOR F. J. LINDERS
PROFESSOR JAMES HARVEY ROGERS
PROFESSOR HENRY SCHULTZ
PROFESSOR WLADYSLAW ZAWADZKI

GROWTH AND DISTRIBUTION OF MEMBERSHIP IN THE ECONOMETRIC SOCIETY AND OF SUBSCRIBERS TO ECONOMETRICA



GEOGRAPHIC DISTRIBUTION OF MEMBERS AND SUBSCRIBERS

	Members	Subscribers	Total		Members	Subscribers	Total
United States	250	130	380	Finland	3	2	5
England	53	21	74	Chile	2	2	4
Italy	42	21	63	Brazil	2	1	3
France	49	10	59	Yugoslavia	3	....	3
Germany	25	12	37	Greece	....	2	2
Netherlands	31	5	36	Mexico	1	1	2
Japan	13	19	32	Neth. India	1	1	2
Norway	23	3	26	Spain	2	....	2
Poland	16	8	24	Uruguay	1	1	2
Switzerland	20	3	23	Algeria	1	....	1
Hungary	19	2	21	Belg. Congo	1	....	1
Belgium	14	4	18	Burma	1	....	1
Sweden	13	5	18	Cuba	....	1	1
Denmark	14	1	15	El Salvador	1	....	1
India	6	9	15	Estonia	1	....	1
Argentina	7	7	14	Latvia	....	1	1
China	10	2	12	Lithuania	....	1	1
Australia	5	6	11	Philippines	1	....	1
Czecho-slovakia	10	....	10	Puerto Rico	....	1	1
U. S. S. R.	....	9	9	Slovakia	1	....	1
Egypt	5	3	8	Straits S.	....	1	1
Canada	3	4	7	Tasmania	....	1	1
Rumania	6	1	7	Venezuela	1	....	1
Bulgaria	5	1	6	Wales	....	1	1
Scotland	5	1	6				
South Africa	4	2	6	Total	671	306	977

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